Quantum electrodynamics in the presence of dielectrics and conductors. VI. Theory of Lippmann fringes

G. S. Agarwal*
Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay-400005, India
(Received 4 February 1975)

A quantum-theoretic treatment of Lippmann fringes is given using the response-function formalism of part I of this series of papers. It is shown that the field emitted by the excited atom in the presence of a dielectric mirror possesses first-order coherence, and the absorption of such a field by the unexcited atom leads to the interference fringes. The nature of the emitted field is also examined from the point of view of excitation of surface modes. The causal character of the absorption process is also briefly commented upon.

Fermi in his classic paper[1] showed how Lippmann fringes can be explained in terms of the quantum theory of radiation. Lippmann fringes are produced when an atom (referred to as atom A) located near a mirror absorbs the radiation produced by an excited atom (referred to as atom B) far from the mirror. Classically, such fringes can be explained easily since atom A essentially sees the standing-wave field formed due to the reflection of incident plane waves. Incident waves can be regarded as homogeneous plane waves since the excited atom is far from the mirror. In Fermi's treatment, the mirror was regarded as a perfect conductor, and hence the radiation field was quantized in the semi-infinite domain bounded by the mirror at $z=0$. The purpose of this short paper is to rederive Fermi's result and present several generalizations using the response functions of paper I and some of the results on spontaneous emission from paper[2] IV. These generalizations are: (a) we describe the mirror by a dielectric function $\epsilon_0(\omega)$ with the dispersion of ϵ_0 included and (b) the atom need not be in the far zone. We examine the field produced by atom B from the point of view of excitation of surface modes. It is also shown that atom A is interacting with a field which has first-order coherence.[3] In our treatment of Lippmann fringes, the role of the field coherence is quite apparent. It should be noted that the process of absorption by atom A of the field emitted by atom B is a second-order process, i.e., the probability of excitation of A is proportional to the fourth power of the electronic charge. We will throughout this paper consider only the dipole transitions.

We have studied in paper IV the spontaneous emission from an atom in the vicinity of a dielectric. We will first recall a few results from that paper, and then we shall establish the coherence character of the field emitted by the atom. The positive-frequency part of the electric field operator is given by [Eq. (IV 5.10)]

$$E_\beta^{(+)}(\vec{r},t)=E_{0\beta}^{(+)}(\vec{r},t)+\frac{i}{\pi}\int_0^t d\tau[S^+(\tau)+S^-(\tau)]\sum_\alpha d_{B\alpha}\int_0^\infty d\omega\,\chi''_{\beta\alpha EE}(\vec{r},\vec{b},\omega)e^{-i\omega(t-\tau)}\,, \tag{1}$$

where we have assumed that our two-level atom described by the pseudospin operators is located at the point $\vec{b}$, and $\vec{d}_B$ is the dipole matrix element connecting the two states of the atom B. The field given by Eq. (1) is exact, i.e., no approximation has been made as to the strength of the interaction. In the far zone and in the long-time approximation, Eq. (1) can be approximated by [Eq. (IV 6.5)]

$$E_\beta^{(+)}(\vec{r},t)\approx E_{0\beta}^{(+)}(\vec{r},t)+\sum_\alpha d_{B\alpha}\tilde{\chi}_{\beta\alpha EE}(\vec{r},\vec{b},\omega_B)S^-\,, \tag{2}$$

where $\tilde{\chi}$ denotes the asymptotic value of χ in the far zone, and ω_B denotes the energy separation between the two levels of the atom. The field as given by Eq. (1) is not an analytic signal,[4] whereas the asymptotic field of Eq. (2) is; i.e., the asymptotic field $\vec{E}^{(+)}$ has contributions only from positive frequencies. The two-time correlation function of the dipole moment operators is given by [Eq. (IV 6.19)]

$$\langle S^+(t)S^-(t')\rangle=\exp[-\gamma_B(t+t')+i\omega_B(t-t')]\,, \tag{3}$$

where γ_B denotes the damping of the two-level atom. On making the rotating-wave approximation, it is clear from Eqs. (1) and (3) that the normally ordered correlation function of the electric field operator is given by

$$\langle E_\alpha^{(-)}(\vec{r}_1, t_1) E_\beta^{(+)}(\vec{r}_2, t_2)\rangle$$

$$= \frac{1}{\pi^2}\sum_{\mu,\nu} d_{B\mu} d_{B\nu} \int_0^{t_1} d\tau_1 \int_0^{t_2} d\tau_2 \iint_0^\infty d\omega_1\, d\omega_2\, \exp[-\gamma_B(\tau_1+\tau_2) + i\omega_B(\tau_1-\tau_2) + i\omega_1(t_1-\tau_1) - i\omega_2(t_2-\tau_2)]$$
$$\times \chi''_{\alpha\mu EE}(\vec{r}_1, \vec{b}, \omega_1)\chi''_{\beta\nu EE}(\vec{r}_2, \vec{b}, \omega_2)$$
$$= V_\alpha^*(\vec{r}_1, t_1) V_\beta(\vec{r}_2, t_2), \quad (4)$$

where

$$V_\beta(\vec{r}, t) = \frac{1}{\pi}\sum_\nu d_{B\nu}\int_0^t d\tau \int_0^\infty d\omega \exp[-\gamma_B\tau - i\omega_B\tau - i\omega(t-\tau)]\chi''_{\beta\nu EE}(\vec{r}, \vec{b}, \omega). \quad (5)$$

A radiation field which satisfies property (4) is said to possess first-order coherence.[3] The field, however, has no higher-order coherence, as there is only one photon present. Hence, atom A will interact with a field which is coherent only to first order. This is to be contrasted with the result found in paper III that a classical field produced by external charges and currents leads to a coherent state of the radiation field, i.e., to a field which is coherent to all orders. The field produced by atom B is thus quantum mechanical in nature. The Laplace transform of Eq. (5) is

$$V_\beta(\vec{r}, p) = \frac{1}{\pi}(p + \gamma_B + i\omega_B)^{-1} \times \sum_\nu d_{B\nu}\int_0^\infty d\omega (p + i\omega)^{-1}\chi''_{\beta\nu EE}(\vec{r}, \vec{b}, \omega),$$
$$\mathrm{Re}\, p \geq 0, \quad (6)$$

where p denotes the Laplace variable.

The explicit form of V_β depends on the nature of the dielectric and the geometrical arrangement. For the case of a dielectric occupying the domain $z \leq 0$, the response functions[5] are given by Eqs. (IV 3.6) and (IV 3.7). We assume here for simplicity that both $\vec{r}$ and $\vec{b}$ are along the z axis; then the nonvanishing response functions (for $z \geq 0$) are

$$\chi_{iiEE}(\vec{r}, \vec{b}, \omega) = \chi^{(0)}_{iiEE}(\vec{r}, \vec{b}, \omega) + \chi^{(1)}_{iiEE}(\vec{r}, \vec{b}, \omega),$$

$$\chi^{(0)}_{iiEE}(\vec{r}, \vec{b}, \omega) = \frac{i}{2\pi}\iint \frac{du\, dv}{w}\hat{\chi}^{(0)}_{ii}(u, v, \omega) e^{iw|z-b|},$$

$$\chi^{(1)}_{iiEE}(\vec{r}, \vec{b}, \omega) = -\frac{i}{2\pi}\iint \frac{du\, dv}{w}\hat{\chi}^{(1)}_{ii}(u, v, \omega) e^{iw(z+b)},$$

$$\hat{\chi}^{(0)}_{ii}(u, v, \omega) = (k_0^2 - k_i^2),$$

$$\hat{\chi}^{(1)}_{11}(u, v, \omega) = \hat{\chi}^{(1)}_{22}(v, u, \omega) = -k_0^2\frac{w - w_0}{w + w_0} - u^2\left(1 - \frac{2w}{w\epsilon_0 + w_0}\right),$$

$$\hat{\chi}^{(1)}_{33}(u, v, \omega) = -(u^2 + v^2)\frac{w\epsilon_0 - w_0}{w\epsilon_0 + w_0},$$

$$k_0 = \omega/c, \quad w^2 = k_0^2 - u^2 - v^2, \quad w_0^2 = k_0^2\epsilon_0 - u^2 - v^2,$$
$$\vec{k} = (u, v, w). \quad (7)$$

The response functions contain contributions both from homogeneous and evanescent waves, since for $u^2 + v^2 > k_0^2$, w is pure imaginary. The response functions contain a resonant denominator $(w\epsilon_0 + w_0)$, the vanishing of which gives dispersion relations for surface modes.[6] Such modes are possible only if the dielectric medium is surface active, i.e., if ϵ_0 takes on negative values. In the present problem, excitation of such modes also takes place due to the evanescent waves emitted by the atom. The contribution to V_β due to the excitation of surface modes can be written as

$$V^{(S)}_\beta(\vec{r}, p) = \frac{1}{\pi}(p + \gamma_B + i\omega_B)^{-1} \times \sum_\nu d_{B\nu}\int d\omega\, (p + i\omega)^{-1}\chi''^{(S)}_{\beta\nu EE}(\vec{r}, \vec{b}, \omega), \quad (8)$$

where the integration is over those frequencies for which $\epsilon_0(\omega) \leq -1$ [since it is known[6] that for the geometrical arrangement considered here, surface modes occur only for frequencies such that $\epsilon_0(\omega) \leq -1$], and where $\chi''^{(S)}$ denotes the surface polariton contribution to χ''. For a simple model of ϵ_0 such as $\epsilon_0 = 1 - \omega_p^2/\omega^2$, the integration in Eq. (8) will thus be over frequencies $\omega \leq \omega_p/\sqrt{2}$. The surface polariton contribution to χ'' is easily obtained from Eq. (7) by using the properties of the resonant denominator, and is found to be [cf. Eq. (IV 9.6)]

$$\chi''^{(S)}_{xxEE} = \chi''^{(S)}_{yyEE} = (1/2|\epsilon_0|)\chi''^{(S)}_{zzEE} = \pi k_0^3\kappa_0^4(|\epsilon_0| + 1)^{-1}(|\epsilon_0| - 1)^{-1/2} \times \exp[-2k_0\sigma_0(b + z)], \quad (9)$$

where κ_0 and σ_0 are defined by the dispersion relation

$$\kappa_0^2 = \epsilon_0/(\epsilon_0 + 1), \quad \sigma_0^2 = \kappa_0^2 - 1 = -1/(\epsilon_0 + 1). \quad (10)$$

On substituting Eq. (9) in Eq. (8) we will obtain the surface polariton field. It is obvious from Eq. (9) that such fields exist only in the close neighborhood of the surface.

We now comment briefly on the causal character of the fields emitted by the atom. For the case of a perfect conductor (letting the conductivity approach infinity, i.e., $\epsilon_0 \to \infty$), the response functions (7) simplify considerably. Then on taking the Fourier transform, one finds that the space and time dependence of the response function is given by

$$\chi_{ijEE}(\vec{r},\vec{r}',t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\, \chi_{ijEE}(\vec{r},\vec{r}',\omega)e^{-i\omega t}$$

$$=\left(-\delta_{ij}\frac{\partial^2}{\partial(ct)^2}+\frac{\partial^2}{\partial x_i\partial x_j}\right)\frac{\delta(t-R/c)}{R}+\left(\delta_{ij}(1-2\delta_{j3})\frac{\partial^2}{\partial(ct)^2}+\frac{\partial^2}{\partial x_i\partial x_j'}\right)\frac{\delta(t-R'/c)}{R'}$$

$$=\chi^{(0)}_{ijEE}(\vec{r},\vec{r}',t)+\chi^{(1)}_{ijEE}(\vec{r},\vec{r}',t),\qquad \vec{R}=\vec{r}-\vec{r}',\quad \vec{R}'=x-x',\quad y-y',\quad z+z', \tag{11}$$

which clearly displays the causal character of the response functions. The first term in Eq. (11) is nonzero only for times $t \geq R/c$, whereas the second term is nonzero for $t\geq R'/c$. Thus, both terms will contribute for $t \geq R'/c$. We recall that Eq. (11) can be given a simple image-charge interpretation. The second term essentially gives the field produced by a dipole placed at the image position $(x',y',-z')$. The dipole moment of the image dipole will have components $(-p_x,-p_y,p_z)$, where the p_i's are the components of the dipole placed at (x',y',z'). We rewrite Eq. (6) as

$$V_\beta(\vec{r},p)=-i\sum_\nu d_{B\nu}(p+\gamma_B+i\omega_B)^{-1}\left(\chi_{\beta\nu EE}(\vec{r},\vec{b},ip)-\frac{1}{\pi}\int_0^\infty d\omega(\omega+ip)^{-1}\chi''_{\beta\nu EE}(\vec{r},\vec{b},\omega)\right),\quad \mathrm{Re}p\to 0,$$

and now it can be shown [cf. our discussion following Eq. (IV 6.4)] that the second term in the above equation makes a negligible contribution in the far zone, and hence the field V_β in the far zone can be approximated by

$$V_\beta(\vec{r},p)\approx -i\sum_\nu d_{B\nu}(p+\gamma_B+i\omega_B)^{-1}\tilde{\chi}_{\beta\nu EE}(\vec{r},\vec{b},ip), \tag{12}$$

where $\tilde{\chi}$ denotes the asymptotic value of χ. The causal character of the field acting on atom A located at $r=a$ will follow on combining Eqs. (11) and (12). Atom A will interact with the field V_β only for times $t\geq R/c$; however, for $R/c\leq t\leq R'/c$, only the direct term $\chi^{(0)}$ acts; i.e., the effect of the dielectric will not be felt. It is only for $t \geq R'/c$, that both terms in Eq. (11) will make a contribution. For such times, Eq. (12) can be further approximated by

$$V_\beta(\vec{r},p)\approx -i\sum_\nu d_{B\nu}(p+\gamma_\beta+i\omega_B)^{-1}\tilde{\chi}_{\beta\nu EE}(\vec{r},\vec{b},\omega_B). \tag{13}$$

In the general case, one must be careful in using the causality concept which is essentially based on the velocity of propagation of the electromagnetic fields. The concept of velocity has unambiguous meaning only for waves of a simple type, such as plane waves (cf. Ref. 4, p. 11). In the asymptotic region it can be shown that χ is essentially a plane wave. To show this, one can use the asymptotic expansion of the angular spectrum of plane waves to obtain the results (recall that in this asymptotic expansion one lets the distance R go to infinity in a fixed direction)

$$\tilde{\chi}^{(0)}_{ijEE}(\vec{r},\vec{r}',\omega)=\frac{e^{ik_0R}}{R}\hat{\chi}^{(0)}_{ij}(k_0\sin\theta\cos\varphi,k_0\sin\theta\sin\varphi,\omega),$$

$$\tilde{\chi}^{(1)}_{ijEE}(\vec{r},\vec{r}',\omega)=-\frac{e^{ik_0R'}}{R'}\hat{\chi}^{(1)}_{ij}(k_0\sin\theta'\cos\varphi',k_0\sin\theta'\sin\varphi',\omega), \tag{14}$$

where θ,φ (θ',φ') are the polar coordinates of the vector $\vec{R}$ $(\vec{R}')$. Hence we will speak of causality only in the asymptotic region. It is obvious from

Eq. (14) that

$$\tilde{\chi}^{(0)} \neq 0 \quad \text{for } t \geq R/c, \qquad \tilde{\chi}^{(1)} \neq 0 \quad \text{for } t \geq R'/c, \tag{15}$$

and therefore, for $t \geq R'/c$, atom A will see the incident field $[\tilde{\chi}^{(0)}]$ as well as the reflected field $[\tilde{\chi}^{(1)}]$. The approximate expression for the field is obtained on combining Eqs. (13) and (14).

We now calculate the probability that atom A makes a transition to the upper state. We first assume that the transition is of the electric dipole type. We have presented in paper III the general result for the transition probabilities in a nonstationary field such as a coherent field. The expression (III 4.5) remains valid even for transitions in a field which is coherent only to first order. The probability that the atom makes a transition to the upper state in first-order perturbation theory and in the long-time approximation is, from Eq. (III 4.5),

$$P \approx \left| \sum_{\beta} d_{A\beta} V_{\beta}(\vec{a}, -i\omega_A) \right|^2 , \tag{16}$$

which, on using Eq. (13), becomes

$$P = [\gamma_B^2 + (\omega_A - \omega_B)^2]^{-1} \left| \sum_{\alpha\beta} d_{A\beta} d_{B\alpha} \tilde{\chi}_{\beta\alpha EE}(\vec{a}, \vec{b}, \omega_B) \right|^2 , \tag{17}$$

where we assumed that atom A is in the far zone of atom B. The transition probability depends on the orientation of the dipole moments. To simplify our considerations, we assume that the dipole moments of each atom are randomly oriented. On carrying out the averaging over the random orientation of the dipole moments, one reduces Eq. (17) to

$$P = \tfrac{1}{9} |d_A|^2 |d_B|^2 [\gamma_B^2 + (\omega_A - \omega_B)^2]^{-1} \times \sum_{\alpha\beta} |\tilde{\chi}_{\beta\alpha EE}(\vec{a}, \vec{b}, \omega_B)|^2 . \tag{18}$$

We also assume for simplicity that each atom is located on the z axis, and that $b \gg a$. Finally, on using Eqs. (7) and (14), we obtain for the transition probability

$$P = \tfrac{2}{9} k_0^4 |d_A|^2 |d_B|^2 [\gamma_B^2 + (\omega_A - \omega_B)^2]^{-1} b^{-2} F , \tag{19}$$

$$F = (1 - |R|)^2 \{1 + [4|R|/(1 - |R|)^2] \sin^2(x - \delta)\} , \qquad x = k_0 a, \quad k_0 = \omega_B / c , \tag{20}$$

where $|R|$ and δ are related to the classical reflection coefficient

$$R = |R| e^{-2i\delta} = \frac{\epsilon_0^{1/2}(\omega_B) - 1}{\epsilon_0^{1/2}(\omega_B) + 1} . \tag{21}$$

The quantum-theoretical formula (19) has the obvious classical interpretation: the factor F is just the square modulus of the sum of the incident wave $e^{-ik_0 a}$ and the reflected field $(\epsilon_0^{1/2} - 1) e^{ik_0 a} / (\epsilon_0^{1/2} + 1)$ (as obtained from the classical Fresnel formula). The appearance of the Lippmann fringes depends on the oscillatory character of F. Fermi's[1] result is regained by assuming infinite conductivity:

$$P \approx \tfrac{8}{9} k_0^4 |d_A|^2 |d_B|^2 [\gamma_B^2 + (\omega_A - \omega_B)^2]^{-1} b^{-2} \sin^2(k_0 a) . \tag{22}$$

We can also introduce a visibility index (cf. Ref. 4, Sec. 10.4.1) by the relation

$$\mathcal{V} = \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}} = \frac{F_{\max} - F_{\min}}{F_{\max} + F_{\min}} = \frac{2|R|}{1 + |R|^2} . \tag{23}$$

The visibility is unity in Fermi's case and also when the frequency ω_B is in the region where ϵ_0 is negative. In general, F oscillates between $(1 - |R|)^2$ and $(1 + |R|)^2$. The positions of the maxima and minima are given by

$$x_{\min} = \delta + n\pi, \quad x_{\max} = \delta + (n + \tfrac{1}{2})\pi , \qquad \tan 2\delta = -2|\epsilon_0|^{1/2} / (|\epsilon_0| - 1), \quad n = \text{integer}, \tag{24}$$

where we have assumed that ϵ_0 is real. Thus, the position of the minimum and maximum depends on the dielectric function.[7,8] In the special cases, one has for the phase difference δ

$$\delta = 0 \quad \text{for perfect conductor} , \tag{25a}$$

$$= \tfrac{1}{2}\pi \quad \text{for } \epsilon_0(\omega_B) = 0 , \quad \text{longitudinal excitation} , \tag{25b}$$

$$= -\tfrac{1}{4}\pi \quad \text{for } \epsilon_0(\omega_B) = -1 , \quad \text{surface excitation} . \tag{25c}$$

We note in passing that if the dielectric were absent, the probability of exciting atom A would be given by Eq. (18) with χ replaced by $\chi^{(0)}$; i.e.,

$$P \sim \tfrac{1}{9} |d_A|^2 |d_B|^2 [\gamma_B^2 + (\omega_A - \omega_B)^2]^{-1} \sum_{\alpha\beta} |\tilde{\chi}^{(0)}_{\beta\alpha EE}(\vec{a}, \vec{b}, \omega_B)|^2 = 0 \quad \text{for } t < |b - a|/c$$
$$= \tfrac{2}{9} k_0^4 |d_A|^2 |d_B|^2 [\gamma_B^2 + (\omega_A - \omega_B)^2]^{-1} b^{-2} \quad \text{for } t \geq |b - a|/c , \tag{26}$$

which is the standard result of the light propagation in vacuum.[1,9]

We next consider the case when atom A makes a magnetic-dipole type of transition. It is easy to show, following the same procedure which led to Eq. (18), that the probability that atom A makes a magnetic-dipole transition is

$$P \sim \tfrac{1}{9}|d_A|^2 |d_B|^2[\gamma_B^2 + (\omega_A - \omega_B)^2]^{-1} \times \sum_{\alpha\beta} |\tilde{\chi}_{\beta\alpha HE}(\vec{a}, \vec{b}, \omega_B)|^2 . \quad (27)$$

The magnetic field response function $\chi_{\beta\alpha HE}$ is related to $\chi_{\beta\alpha EE}$ by

$$\chi_{\beta\alpha HE}(\vec{r}, \vec{r}\,', \omega) = \frac{1}{ik_0} \sum \epsilon_{\beta\mu\nu} \frac{\partial}{\partial x_\mu} \chi_{\nu\alpha EE}(\vec{r}, \vec{r}\,', \omega) , \quad (28)$$

where $\epsilon_{\beta\mu\nu}$ is the completely antisymmetric tensor. The relation (28) is simply a consequence of the Maxwell equation

$$\vec{H}(\vec{r}, \omega) = (1/ik_0)\nabla \times \vec{E}(\vec{r}, \omega) .$$

In the special case when both the atoms are located on the z axis, we obtain, on combining Eqs. (7), (14), (27), and (28), the result

$$P \sim \tfrac{2}{9} k_0^4 |d_A|^2 |d_B|^2[\gamma_B^2 + (\omega_A - \omega_B)^2]^{-1} b^{-2}(1 - |R|)^2 \times \{1 + [4|R|/(1 - |R|)^2] \cos^2(x - \delta)\} , \quad (29)$$

which again shows the interference fringes. The position of the maxima for the magnetic-dipole case corresponds to the position of the minima for the electric-dipole case, and vice versa.

We close this paper with a few remarks concerning the transition probability of the exciting atom A when atom B is in the near zone of A. This case is usually referred to as the energy-transfer problem,[10] and is usually treated by ignoring retardation. In the treatment given above, we have included the retardation. We now establish the connection of our treatment with the usual treatment.[10] The radiation field emitted by the atom can also be expressed as [cf. Eq. (IV 6.2)]

$$E_\beta^{(+)}(\vec{r}, t) \sim E_{0\beta}^{(+)}(\vec{r}, t) + \sum_\alpha d_{B\alpha} \chi_{\beta\alpha EE}(\vec{r}, \vec{b}, \omega_B) S^- + (S^+ - S^-) \sum_\alpha \frac{d_{B\alpha}}{\pi} \int_0^\infty d\omega\, (\omega + \omega_B)^{-1} \times \chi''_{\beta\alpha EE}(\vec{r}, \vec{b}, \omega) . \quad (30)$$

For the purpose of calculating the transition probability, it is sufficient to work with the total field, which, according to Eq. (30), will be

$$E_\beta(\vec{r}, t) \sim E_{0\beta}(\vec{r}, t) + \sum_\alpha d_{B\alpha}[\chi_{\beta\alpha EE}(\vec{r}, \vec{b}, \omega_B) S^- + \text{H.c.}] . \quad (31)$$

The transition probability will therefore be

$$P \propto \left| \sum_{\alpha\beta} d_{B\alpha} d_{A\beta} \chi_{\beta\alpha EE}(\vec{a}, \vec{b}, \omega_B) \right|^2 \quad (32)$$

$$\approx \left| \sum_{\alpha\beta} d_{B\alpha} d_{A\beta} \chi'_{\beta\alpha EE}(\vec{a}, \vec{b}, \omega_B) \right|^2 , \quad (33)$$

since in the near zone the dominant contribution comes from the real part of $\chi_{\beta\alpha EE}$. It is interesting to note that the surface polariton modes will contribute to the transition probability of Eq. (33). If in Eq. (33) we substitute the free-space response function, then we will obtain the well-known results of the energy transfer.[10] Finally, the result (33) is easily generalized for higher-order multipole transitions.

---

*Present permanent address: Institute of Science, Madame Cama Road, Bombay-400032, India.

[1]E. Fermi, Rev. Mod. Phys. 4, 87 (1932).

[2]G. S. Agarwal, Phys. Rev. A 11, 230, 253 (1975); 12, 1475 (1975); these will be referred to as Paper I, III, and IV, respectively, and equations referring to these papers will carry the corresponding index.

[3]C. L. Mehta, J. Math. Phys. 8, 1798 (1967); R. J. Glauber, Phys. Rev. 130, 2529 (1965).

[4]M. Born and E. Wolf, *Principles of Optics* (Pergamon, Oxford, 1970).

[5]We have recently realized that some of the response functions calculated in I are related, in special cases, to the fields obtained in Sommerfeld's problem, and for a good discussion of this problem we refer the reader to G. Tyras, *Radiation and Propagation of Electromagnetic Waves* (Academic, New York, 1969).

[6]For a review of surface excitations see, e.g., R. H. Ritchie, Surf. Sci. 34, 1 (1973).

[7]We can, in a similar manner, consider the ionization of atom A in the field produced by atom B and the dielectric mirror. We will again obtain interference fringes. This is analogous to ionization in the field of two laser beams, a discussion of which can be found in D. Marcuse, *Engineering Quantum Electrodynamics* (Harcourt, Brace and World, New York, 1970), p. 205.

[8]One can also consider higher-order processes involving the scattering, by atom A, of the radiation emitted by atom B, e.g., Thomson scattering. In the special case this would be the scattering in a standing-wave field,

which is similar to the scattering effect in a Dirac-Kapitza arrangement, cf. J. H. Eberly, *Progress in Optics*, edited by E. Wolf (North-Holland, Amsterdam, 1969), Vol. VII, pp. 379–399; S. Altshuler, L. M. Frantz, and R. Braunstein, Phys. Rev. Lett. 17, 231 (1966).

[9]W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973), p. 314; for alternate treatments of causality in spontaneous emission, we refer the reader to G. S. Agarwal, *Springer Tracts in Modern Physics*, edited by G. Höhler *et al*. (Springer-Verlag, New York, 1974), Vol. 70, p. 38; P. W. Milonni and P. L. Knight, Phys. Rev. A 10, 1096 (1974).

[10]K. H. Drexhage, *Progress in Optics*, edited by E. Wolf (North-Holland, Amsterdam, 1974), p. 163; T. Förster, Ann. Phys. (Leipz.) 6, 55 (1948); D. L. Dexter, J. Chem. Phys. 21, 836 (1953); H. Kuhn, J. Chem. Phys. 53, 101 (1970).