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Quantum electrodynamics in the presence of dielectrics and conductors. I. Electromagnetic-field response functions and black-body fluctuations in finite geometries

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A form of quantum electrodynamics is developed which allows us to treat a number of problems involving dielectric and conducting surfaces, the presence of which leads to a number of new observable effects. A number of suitably defined response functions play a basic role in the present approach, as these in conjunction with the fluctuation-dissipation theorem lead to electromagnetic field correlation functions, which describe physical effects such as lifetimes, frequency shifts of the excited states, dispersion forces, etc. The quantization of the electromagnetic field is only implicitly used. A large part of the present paper is devoted to the calculation of the response functions involving different geometries and various types of dielectrics. Both spatially dispersive and spatially nondispersive dielectrics are considered. The response functions are calculated using Maxwell's equations and the usual boundary conditions at the interface adjoining the two mediums. As a first application of the present approach, the black-body fluctuations in finite geometries and the influence of surfaces on its temporal and spatial coherence are studied. An interesting theorem is also proved which enables us to calculate the normally ordered (antinormally ordered) correlation functions from the symmetrized correlation functions.

I. INTRODUCTION

The interaction of radiation with matter has always been a fascinating subject. Over the last decade it has received a new impetus and a large class of new phenomena have been discovered. Different kinds of theories have been advanced to discuss the general problem of interaction of radiation with matter (cf. Refs. 1, 2). Most of the systems studied to date (except the case of laser emission and some problems in nonlinear optics) correspond to the interaction of the matter with the electromagnetic field in free space. Such systems include the well-known problems of (i) coherence properties of black-body radiation,[3] (ii) absorption and emission of electromagnetic waves,[4] (iii) spontaneous emission free space[1]—both dynamical and kinematical aspects, (iv) anomalous magnetic moment of the electron,[5] etc. Such problems have been treated by quantizing the electromagnetic field in the entire free space and by using appropriate perturbation theory.[1-6]

In the present series we would like to investigate how the presence of the dielectric and conducting surfaces affects, say, lifetimes of excited states, Lamb shifts, the anomalous magnetic moment of the electron, and in general the transition probabilities, coherence properties of the black-body radiation, etc. We will show that the zero-point fluctuations of the dielectric field lead to appreciable effects, which, within our present experimental capability are observable.

To deal with all these problems, we need a form of electrodynamics applicable in presence of dielectric interfaces. One obvious way is first to solve the classical Maxwell equations and then quantize the solutions. However, one runs into problems with such a procedure, and the quantization can be done only in certain specific situations.[7] Fortunately, it so happens that in all the above-mentioned problems, we do not need to quantize the field explicitly, as all the relevant (observable) entities can be shown to be related to the expectation values of the commutators and anticommutators of the field operators at different space-time points.[8] We also know, from response theory[9,10] from statistical mechanics, that such expectation values, for equilibrium systems, can be related to suitably defined response functions. Hence it is clear that the problem of interaction of radiation with matter, in presence of dielectric interfaces, is solved once the appropriate response functions are known. The first part of this series of papers is devoted to the calculation of response functions involving various geometries and different kinds of dielectrics. The plan of this paper is as follows.

We begin in Sec. II by summarizing some results from the linear response theory and discussing the types of external probes to be considered in the calculation of electromagnetic field fluctuations. In Sec. III we prove an important theorem concerning the symmetrized and normally ordered correlation functions. In Sec. IV black-body fluctuations in the entire free space are considered and the results of Mehta and Wolf obtained. This elementary example is included as it serves to illustrate the basic concepts involved. We calculate in Sec. V the electromagnetic field response functions in the region bounded by two isotropic nonmagnetic dielectrics, and in Sec. VI the same calculation is done when one of the dielectrics is replaced by a spatially dispersive dielectric. In Sec. VII, we use the results of Secs. II, III, and V to calculate the coherence properties of black-body fluctuations in *constrained* geometries and show how the presence of a surface affects its temporal and spatial coherence.

II. LINEAR RESPONSE THEORY AND ELECTROMAGNETIC FIELD FLUCTUATIONS

In this section we recall few formulas from linear-response theory and discuss the types of probes needed to describe the electromagnetic field fluctuations both in free space and in finite geometries. Consider a quantum-mechanical system characterized by a Hamiltonian H_0 and the equilibrium density matrix[11]

$$\rho = e^{-\beta H_0}/\mathrm{tr}(e^{-\beta H_0}), \quad \beta = 1/k_B T \tag{2.1}$$

where k_B is the Boltzmann constant and T is temperature. Let us perturb this system by an external perturbation of the form

$$H_{\mathrm{ext}} = -\int d^3r \sum_j A_j(\vec{r}, t) f_j(\vec{r}, t), \tag{2.2}$$

where f_j are the external forces and A_j are the dynamical variables of the system under consideration. A straightforward perturbation theory shows that the linear response of the variable A_i to f_j is given by

$$\delta\langle A_i(\vec{r}, t)\rangle = \sum_j \int d^3r' \times \int dt' \chi_{ij}(\vec{r}, \vec{r}', t-t') f_j(\vec{r}', t'), \tag{2.3}$$

where $\chi_{ij}(\vec{r}, \vec{r}', t-t')$ is the usual susceptibility tensor defined by

$$\chi_{ij}(\vec{r}, \vec{r}', t-t') = 2i\eta(t-t')\chi''_{ij}(\vec{r}, \vec{r}', t-t') \tag{2.4}$$

with

$$\chi_{ij} = \chi'_{ij} + i\chi''_{ij}, \quad \chi''_{ij} = (1/2\hbar)\langle[A_i(\vec{r}, t),\ A_j(\vec{r}', t')]\rangle, \tag{2.5}$$

$$\chi'_{ij}(\vec{r}, \vec{r}', \omega) = \mathrm{P}\int \frac{d\omega'}{\pi} \frac{\chi''_{ij}(\vec{r}, \vec{r}', \omega')}{\omega' - \omega}, \tag{2.6}$$

where η is a step function: $\eta(\tau) = 1$ if $\tau > 0$ and zero otherwise and where $\langle\cdots\rangle$ denotes an average with respect to (2.1). It is clear from (2.3) that

$$\frac{\delta\langle A_i(\vec{r}, \omega)\rangle}{\delta f_j(\vec{r}', \omega)} = \chi_{ij}(\vec{r}, \vec{r}', \omega), \tag{2.7}$$

where the Fourier-transformed quantities are defined by

$$\psi(t-t') = (1/2\pi)\int d\omega\, \psi(\omega) e^{-i\omega(t-t')}. \tag{2.8}$$

The symmetrized correlation function defined by

$$S_{ij}(\vec{r}, \vec{r}', t-t') = \tfrac{1}{2}\langle\{A_i(\vec{r}, t) - \langle A_i(\vec{r}, t)\rangle, A_j(\vec{r}', t') - \langle A_j(\vec{r}', t')\rangle\}\rangle, \tag{2.9}$$

is given by the fluctuation-dissipation theorem

$$S_{ij}(\vec{r}, \vec{r}', \omega) = \hbar \coth(\beta\omega\hbar/2)\chi''_{ij}(\vec{r}, \vec{r}', \omega). \tag{2.10}$$

If the variables A_i and A_j have the same signature under time reversal, then $\chi''_{ij}(\vec{r}, \vec{r}', \omega)$ is odd in ω, real and symmetric under the interchange $i \rightleftharpoons j$, $\vec{r} \rightleftharpoons \vec{r}'$, and hence χ'' is the imaginary part of χ. If A_i and A_j have opposite parity, then $\chi''_{ij}(\vec{r}, \vec{r}', \omega)$ is even in ω, pure imaginary, and antisymmetric under the interchange $i \rightleftharpoons j$, $\vec{r} \rightleftharpoons \vec{r}'$ and therefore in this case $\chi''_{ij} = -i\,\mathrm{Re}\chi_{ij}$. The symmetry properties of $\chi'_{ij}(\vec{r}, \vec{r}', \omega)$ follow from the dispersion relation (2.6).

For the problem of electromagnetic fluctuations the external probes will be taken to be external polarization $\vec{\mathcal{P}}(\vec{r}, t)$ and external magnetization $\vec{\mathfrak{M}}(\vec{r}, t)$. The Hamiltonian H_{ext} in the present case is

$$H_{\mathrm{ext}} = -\int [\vec{\mathcal{P}}(\vec{r}, t)\cdot\vec{\mathrm{E}}(\vec{r}, t) + \vec{\mathfrak{M}}(\vec{r}, t)\cdot\vec{\mathrm{H}}(\vec{r}, t)]\, d^3r, \tag{2.11}$$

where $\vec{\mathrm{E}}$ and $\vec{\mathrm{H}}$ are the second-quantized operators corresponding to the electric and magnetic field, respectively. We now introduce four types of response functions:

$$\chi_{ijEE}(\vec{r}, \vec{r}', \omega) = \delta\langle E_i(\vec{r}, \omega)\rangle/\delta\mathcal{P}_j(\vec{r}', \omega), \tag{2.12}$$

$$\chi_{ijEH}(\vec{r}, \vec{r}', \omega) = \delta\langle E_i(\vec{r}, \omega)\rangle/\delta\mathfrak{M}_j(\vec{r}', \omega), \tag{2.13}$$

$$\chi_{ijHE}(\vec{r}, \vec{r}', \omega) = \delta\langle H_i(\vec{r}, \omega)\rangle/\delta\mathcal{P}_j(\vec{r}', \omega), \tag{2.14}$$

$$\chi_{ijHH}(\vec{r}, \vec{r}', \omega) = \delta\langle H_i(\vec{r}, \omega)\rangle/\delta\mathfrak{M}_j(\vec{r}', \omega), \tag{2.15}$$

and we introduce the corresponding symmetrized correlation functions

$$\mathcal{E}^{(S)}_{ij}(\vec{r}, \vec{r}', t-t') = \tfrac{1}{2}\langle\{E_i(\vec{r}, t), E_j(\vec{r}', t')\}\rangle, \tag{2.16}$$

$$\mathcal{G}^{(S)}_{ij}(\vec{r}, \vec{r}', t-t') = \tfrac{1}{2}\langle\{E_i(\vec{r}, t), H_j(\vec{r}', t')\}\rangle, \tag{2.17}$$

$$\tilde{\mathcal{G}}^{(S)}_{ij}(\vec{r}, \vec{r}', t-t') = \tfrac{1}{2}\langle\{H_i(\vec{r}, t), E_j(\vec{r}', t')\}\rangle, \tag{2.18}$$

$$\mathcal{H}^{(S)}_{ij}(\vec{r}, \vec{r}', t-t') = \tfrac{1}{2}\langle\{H_i(\vec{r}, t), H_j(\vec{r}', t')\}\rangle. \tag{2.19}$$

It should be noted that $\vec{\mathrm{E}}$ ($\vec{\mathrm{H}}$) is an even (odd) variable under time reversal and hence χ''_{ijEE}, χ''_{ijHH} will, respectively, be the imaginary parts of χ_{ijEE}, χ_{ijHH}. Similarly χ''_{ijEH}, χ''_{ijHE} will be equal to $-i\,\mathrm{Re}\chi_{ijEH}$, $-i\,\mathrm{Re}\chi_{ijHE}$. From the fluctuation-dissipation theorem (2.10), it is now clear that

$$\mathcal{E}^{(S)}_{ij}(\vec{r}, \vec{r}', \omega) = \hbar \coth(\beta\omega\hbar/2)\,\mathrm{Im}\chi_{ijEE}(\vec{r}, \vec{r}', \omega), \tag{2.20}$$

$$\mathcal{G}_{ij}^{(S)}(\vec{r},\vec{r}',\omega) = -i\hbar\coth(\beta\omega\hbar/2)\mathrm{Re}\chi_{ijEH}(\vec{r},\vec{r}',\omega), \tag{2.21}$$

$$\tilde{\mathcal{G}}_{ij}^{(S)}(\vec{r},\vec{r}',\omega) = -i\hbar\coth(\beta\omega\hbar/2)\mathrm{Re}\chi_{ijHE}(\vec{r},\vec{r}',\omega), \tag{2.22}$$

$$\mathcal{H}_{ij}^{(S)}(\vec{r},\vec{r}',\omega) = \hbar\coth(\beta\omega\hbar/2)\mathrm{Im}\chi_{ijHH}(\vec{r},\vec{r}',\omega); \tag{2.23}$$

thus the four response functions defined by (2.12)–(2.15) completely determine the correlation functions. The response functions are to be calculated from the solution of Maxwell's equations:

$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}(\vec{B}+4\pi\vec{\mathfrak{M}}),\quad \nabla\cdot(\vec{B}+4\pi\vec{\mathfrak{M}})=0, \tag{2.24}$$

$$\nabla\times\vec{H} = \frac{1}{c}\frac{\partial}{\partial t}(\vec{D}+4\pi\vec{\mathcal{P}}),\quad \nabla\cdot(\vec{D}+4\pi\vec{\mathcal{P}})=0. \tag{2.25}$$

Equations (2.24), (2.25) for the response are to be solved subject to the usual boundary conditions, namely (i) tangential components of $\vec{E}$ and $\vec{H}$ and (ii) normal components of $\vec{D}$ and $\vec{B}$ are continuous across an interface, where $\vec{D}$ and $\vec{B}$ as usual denote the electric induction and magnetic induction, respectively. We, of course, assume that no surface currents and surface charges are present. We will describe the dielectric in terms of the appropriate dielectric function. This is the only phenomenological element in the theory. We will also assume "sharp" surfaces.

III. RELATION BETWEEN SYMMETRIZED CORRELATION FUNCTIONS AND THE NORMALLY ORDERED CORRELATION FUNCTIONS

The fluctuation dissipation theorem enables us to calculate the symmetrized correlation functions of the form (2.16)–(2.19) in terms of linear response. In many applications such as in photon absorption measurements,[4] one measures normally ordered correlations of the form

$$\langle\vec{E}^{(-)}(\vec{r},t)\vec{E}^{(+)}(\vec{r}',t')\rangle,$$

where $\vec{E}^{(+)}$ and $\vec{E}^{(-)}$ are the positive and negative frequency parts of the field operator $\vec{E}$. In this section we discuss the relation which exists between two types of correlation functions.

We assume that the electromagnetic fields are represented by analytic signals[12]; then their positive and negative frequency parts are defined by

$$E_j^{(\pm)} = \tfrac{1}{2}(E_j \pm i\tilde{E}_j),\quad H_j^{(\pm)} = \tfrac{1}{2}(H_j \pm i\tilde{H}_j), \tag{3.1}$$

where $\tilde{E}_j(\vec{r},t)$ $[\tilde{H}_j(\vec{r},t)]$ is given in terms of $E_j(\vec{r},t)$ $[H_j(\vec{r},t)]$ by the Hilbert transform relation

$$\tilde{E}_j(\vec{r},t) = \frac{1}{\pi}\mathrm{P}\int_{-\infty}^{+\infty}\frac{E_j(\vec{r},t')dt'}{t'-t},\quad \tilde{H}_j(\vec{r},t) = \frac{1}{\pi}\mathrm{P}\int_{-\infty}^{+\infty}\frac{H_j(\vec{r},t')dt'}{t'-t}. \tag{3.2}$$

We introduce normally ordered correlation functions:

$$\mathcal{E}_{ij}^{(N)}(\vec{r},\vec{r}',t-t') = \langle E_i^{(-)}(\vec{r},t)E_j^{(+)}(\vec{r}',t')\rangle, \tag{3.3}$$

$$\mathcal{G}_{ij}^{(N)}(\vec{r},\vec{r}',t-t') = \langle E_i^{(-)}(\vec{r},t)H_j^{(+)}(\vec{r}',t')\rangle, \tag{3.4}$$

$$\tilde{\mathcal{G}}_{ij}^{(N)}(\vec{r},\vec{r}',t-t') = \langle H_i^{(-)}(\vec{r},t)E_j^{(+)}(\vec{r}',t')\rangle, \tag{3.5}$$

$$\mathcal{H}_{ij}^{(N)}(\vec{r},\vec{r}',t-t') = \langle H_i^{(-)}(\vec{r},t)H_j^{(+)}(\vec{r}',t')\rangle. \tag{3.6}$$

It should be noted that we are dealing with stationary fields, i.e., fields for which many-time correlation functions are invariant under time translation. For such fields, (3.2) leads to interesting relations[13] between the correlation functions of E and $\tilde{E}$. For example, we have from (3.2)

$$\begin{aligned}\langle\tilde{E}_i(\vec{r}_1,t_1)\tilde{E}_j(\vec{r}_2,t_2)\rangle &= \frac{1}{\pi^2}\mathrm{P}\int\int\frac{dt'\,dt''\langle E_i(\vec{r}_1,t')E_j(\vec{r}_2,t'')\rangle}{(t'-t_1)(t''-t_2)}\\ &= \frac{1}{\pi^2}\mathrm{P}\int\int dt'\,dt''\frac{\langle E_i(\vec{r}_1,t'-t'')E_j(\vec{r}_2,0)\rangle}{(t'-t_1)(t''-t_2)}\\ &= \frac{1}{\pi^2}\mathrm{P}\int\int dt'\,dt''\frac{\langle E_i(\vec{r}_1,t')E_j(\vec{r}_2,0)\rangle}{(t''-t_2)(t''-t_1+t')},\end{aligned}$$

which on using the identity

$$\frac{1}{\pi^2}\mathrm{P}\int\frac{dt''}{(t''-t)(t''-\tau)} = \delta(t-\tau)$$

becomes

$$\langle\tilde{E}_i(\vec{r}_1,t_1)\tilde{E}_j(\vec{r}_2,t_2)\rangle = \langle E_i(\vec{r}_1,t_1-t_2)E_j(\vec{r}_2,0)\rangle$$

and hence

$$\langle\tilde{E}_i(\vec{r}_1,t_1)\tilde{E}_j(\vec{r}_2,t_2)\rangle = \langle E_i(\vec{r}_1,t_1)E_j(\vec{r}_2,t_2)\rangle. \tag{3.7}$$

Similarly one can prove that

$$\langle E_j(\vec{r}_1,t_1)\tilde{E}_i(\vec{r}_2,t_2)\rangle = -\langle\tilde{E}_j(\vec{r}_1,t_1)E_i(\vec{r}_2,t_2)\rangle. \tag{3.8}$$

Hence from (3.7) and (3.8) it follows that

$$\begin{aligned}\mathcal{E}_{ij}^{(N)}(\vec{r}_1,\vec{r}_2,t_1-t_2) &= \tfrac{1}{2}\langle E_i(\vec{r}_1,t_1)E_j(\vec{r}_2,t_2)\rangle + \tfrac{1}{2}i\langle E_i(\vec{r}_1,t_1)\tilde{E}_j(\vec{r}_2,t_2)\rangle\\ &= \frac{1}{2\pi}\int_{-\infty}^{+\infty}dt''\,\delta_+(t''-t_2)\times\langle E_i(\vec{r}_1,t_1)E_j(\vec{r}_2,t'')\rangle\\ &= \frac{1}{2\pi}\int_{-\infty}^{+\infty}dt''\,\delta_+(t_1-t_2-t'')\times\langle E_i(\vec{r}_1,t'')E_j(\vec{r}_2,0)\rangle.\end{aligned} \tag{3.9}$$

Note that from (2.5) and (2.16) one has

$$\langle E_i(\vec{r}_1, t'') E_j(\vec{r}_2, 0)\rangle = \hbar\chi''_{ijEE}(\vec{r}_1, \vec{r}_2, t'') + \mathcal{E}^{(S)}_{ij}(\vec{r}_1, \vec{r}_2, t'')$$

hence

$$\mathcal{E}^{(N)}_{ij}(\vec{r}_1, \vec{r}_2, t_1 - t_2) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dt''\, \delta_+(t_1 - t_2 - t'') \times [\mathcal{E}^{(S)}_{ij}(\vec{r}_1, \vec{r}_2, t'') + \hbar\chi''_{ijEE}(\vec{r}_1, \vec{r}_2, t'')]. \quad (3.10)$$

On taking the Fourier transform, (3.10) leads to

$$\mathcal{E}^{(N)}_{ij}(\vec{r}_1, \vec{r}_2, \omega) = \eta(-\omega)[\hbar\chi''_{ijEE}(\vec{r}_1, \vec{r}_2, \omega) + \mathcal{E}^{(S)}_{ij}(\vec{r}_1, \vec{r}_2, \omega)], \quad (3.11)$$

which is the desired relation between $\mathcal{E}^{(N)}$ and $\mathcal{E}^{(S)}$. In obtaining (3.11) we used the relation

$$\eta(-\omega) = (1/2\pi)\int_{-\infty}^{+\infty} e^{i\omega\tau}\delta_+(\tau)\, d\tau. \quad (3.12)$$

On using (2.20), (3.11) becomes

$$\mathcal{E}^{(N)}_{ij}(\vec{r}_1, \vec{r}_2, \omega) = \hbar\eta(-\omega)[1 + \coth(\beta\omega\hbar/2)] \times \mathrm{Im}\chi_{ijEE}(\vec{r}_1, \vec{r}_2, \omega). \quad (3.13)$$

Similarly one can show that

$$\mathcal{H}^{(N)}_{ij}(\vec{r}_1, \vec{r}_2, \omega) = \hbar\eta(-\omega)[1 + \coth(\beta\omega\hbar/2)] \times \mathrm{Im}\chi_{ijHH}(\vec{r}_1, \vec{r}_2, \omega), \quad (3.14)$$

$$\mathcal{G}^{(N)}_{ij}(\vec{r}_1, \vec{r}_2, \omega) = -i\hbar\eta(-\omega)[1 + \coth(\beta\omega\hbar/2)] \times \mathrm{Re}\chi_{ijEH}(\vec{r}_1, \vec{r}_2, \omega), \quad (3.15)$$

$$\tilde{\mathcal{G}}^{(N)}_{ij}(\vec{r}_1, \vec{r}_2, \omega) = -i\hbar\eta(-\omega)[1 + \coth(\beta\omega\hbar/2)] \times \mathrm{Re}\chi_{ijHE}(\vec{r}_1, \vec{r}_2, \omega). \quad (3.16)$$

Relations (3.13)–(3.16) are very basic and show how the normally ordered correlation functions can be obtained from the knowledge of the linear response functions. It is interesting to note that the Fourier transforms of the normally ordered correlations vanish for positive frequencies with Fourier transforms defined by (2.8).

If the measurements are carried with a quantum counter,[14] then we need to calculate the antinormally ordered correlations defined by

$$\mathcal{E}^{(A)}_{ij}(\vec{r}_1, \vec{r}_2, t_1 - t_2) = \langle E^{(+)}_i(\vec{r}_1, t_1) E^{(-)}_j(\vec{r}_2, t_2)\rangle, \quad (3.17)$$

$$\mathcal{G}^{(A)}_{ij}(\vec{r}_1, \vec{r}_2, t_1 - t_2) = \langle E^{(+)}_i(\vec{r}_1, t_1) H^{(-)}_j(\vec{r}_2, t_2)\rangle, \quad (3.18)$$

$$\tilde{\mathcal{G}}^{(A)}_{ij}(\vec{r}_1, \vec{r}_2, t_1 - t_2) = \langle H^{(+)}_i(\vec{r}_1, t_1) E^{(-)}_j(\vec{r}_2, t_2)\rangle, \quad (3.19)$$

$$\mathcal{H}^{(A)}_{ij}(\vec{r}_1, \vec{r}_2, t_1 - t_2) = \langle H^{(+)}_i(\vec{r}_1, t_1) H^{(-)}_j(\vec{r}_2, t_2)\rangle. \quad (3.20)$$

These can again be related to symmetrized correlations; for example, in place of (3.10) we have

$$\mathcal{E}^{(A)}_{ij}(\vec{r}_1, \vec{r}_2, t_1 - t_2) = (1/2\pi)\int_{-\infty}^{+\infty} dt''\, \delta_-(t_1 - t_2 - t'') \times [\hbar\chi''_{ijEE}(\vec{r}_1, \vec{r}_2, t'') + \mathcal{E}^{(S)}_{ij}(\vec{r}_1, \vec{r}_2, t'')], \quad (3.21)$$

and hence

$$\mathcal{E}^{(A)}_{ij}(\vec{r}_1, \vec{r}_2, \omega) = \eta(\omega)\hbar[1 + \coth(\beta\omega\hbar/2)] \times \mathrm{Im}\chi_{ijEE}(\vec{r}_1, \vec{r}_2, \omega), \quad (3.22)$$

$$\mathcal{G}^{(A)}_{ij}(\vec{r}_1, \vec{r}_2, \omega) = -i\eta(\omega)\hbar[1 + \coth(\beta\omega\hbar/2)] \times \mathrm{Re}\chi_{ijEH}(\vec{r}_1, \vec{r}_2, \omega), \quad (3.23)$$

$$\tilde{\mathcal{G}}^{(A)}_{ij}(\vec{r}_1, \vec{r}_2, \omega) = -i\eta(\omega)\hbar[1 + \coth(\beta\omega\hbar/2)] \times \mathrm{Re}\chi_{ijHE}(\vec{r}_1, \vec{r}_2, \omega), \quad (3.24)$$

$$\mathcal{H}^{(A)}_{ij}(\vec{r}_1, \vec{r}_2, \omega) = \eta(\omega)\hbar[1 + \coth(\beta\omega\hbar/2)] \times \mathrm{Im}\chi_{ijHH}(\vec{r}_1, \vec{r}_2, \omega), \quad (3.25)$$

showing that the antinormally ordered correlation functions only have positive frequency components. The relations of this section will be useful in Secs. IV and VII of the present paper and in Paper III of the present series of papers.

IV. BLACK-BODY FLUCTUATIONS IN INFINITE DOMAIN

As a preliminary application of the results of Secs. II and III, we consider black-body fluctuations in infinite domain. These have been extensively discussed by Mehta and Wolf.[3] We include this example just to show how the response functions can be used. Mehta and Wolf took the usual expansion of the field operators and used the diagonal coherent-state representation[4] of the density operator to calculate the normally ordered correlation functions.

From the Maxwell equations (2.24) and (2.25) we obtain

$$\nabla \times \nabla \times \vec{E} = ik_0[-ik_0\vec{E} + 4\pi\nabla\times\vec{\mathfrak{M}} - ik_0 4\pi\vec{\mathcal{P}}],$$

$$\nabla \cdot \vec{E} = -4\pi\nabla\cdot\vec{\mathcal{P}}, \quad k_0 = \omega/c,$$

which can be rewritten as

$$(\nabla^2 + k_0^2)\vec{E} = -4\pi[k_0^2\vec{\mathcal{P}} + \vec{\nabla}(\vec{\nabla}\cdot\vec{\mathcal{P}})] - 4\pi i k_0 \nabla\times\vec{\mathfrak{M}}. \quad (4.1)$$

Similarly we find

$$(\nabla^2 + k_0^2)\vec{H} = -4\pi[k_0^2\vec{\mathfrak{M}} + \vec{\nabla}(\vec{\nabla}\cdot\vec{\mathfrak{M}})] + 4\pi i k_0 \nabla\times\vec{\mathcal{P}}. \quad (4.2)$$

These equations should be solved subject to the outgoing boundary conditions at infinity. We assume, throughout this paper, external probes of the form

$$\vec{\mathcal{P}}(\vec{r},\omega)=\vec{p}(\omega)\delta(\vec{r}-\vec{r}_0),\quad \vec{\mathfrak{M}}(\vec{r},\omega)=\vec{m}(\omega)\delta(\vec{r}-\vec{r}_0). \tag{4.3}$$

Let G_0 be the free-space Green's function

$$G_0(|\vec{r}-\vec{r}'|)=e^{ik_0|\vec{r}-\vec{r}'|}/|\vec{r}-\vec{r}'|, \tag{4.4}$$

then the solution of (4.1) and (4.2) is

$$\vec{E}(\vec{r},\omega)=[k_0^2\vec{p}+\vec{\nabla}(\vec{p}\cdot\vec{\nabla})+ik_0\nabla\times\vec{m}]\times e^{ik_0|\vec{r}-\vec{r}_0|}/|\vec{r}-\vec{r}_0|, \tag{4.5}$$

$$\vec{H}(\vec{r},\omega)=[k_0^2\vec{m}+\vec{\nabla}(\vec{m}\cdot\vec{\nabla})-ik_0\nabla\times\vec{p}]\times e^{ik_0|\vec{r}-\vec{r}_0|}/|\vec{r}-\vec{r}_0|. \tag{4.6}$$

It is now clear from (4.3), (4.5), (4.6) and the defining relations (2.7), (2.12)–(2.15) that

$$\chi_{ijEE}(\vec{r},\vec{r}',\omega)=\chi_{ijHH}(\vec{r},\vec{r}',\omega)=\left(k_0^2\delta_{ij}+\frac{\partial^2}{\partial x_i\partial x_j}\right)G_0(\vec{r}-\vec{r}'), \tag{4.7}$$

$$\chi_{ijEH}(\vec{r},\vec{r}',\omega)=-\chi_{ijHE}(\vec{r},\vec{r}',\omega)=-ik_0\epsilon_{ijl}\frac{\partial}{\partial x_l}G_0(\vec{r}-\vec{r}'), \tag{4.8}$$

where ϵ_{ijk} is the completely antisymmetric tensor of Levicivita. Hence on combining (2.20)–(2.23), (4.7), and (4.8) we obtain the symmetrized correlation functions

$$\mathcal{E}_{ij}^{(S)}(\vec{r},\vec{r}',\omega)=\mathcal{H}_{ij}^{(S)}(\vec{r},\vec{r}',\omega)=\hbar\coth\left(\frac{\beta\omega\hbar}{2}\right)\left(k_0^2\delta_{ij}+\frac{\partial^2}{\partial x_i\partial x_j}\right)\frac{\sin(k_0|\vec{r}-\vec{r}'|)}{|\vec{r}-\vec{r}'|}, \tag{4.9}$$

$$\mathcal{G}_{ij}^{(S)}(\vec{r},\vec{r}',\omega)=-\tilde{\mathcal{G}}_{ij}^{(S)}(\vec{r},\vec{r}',\omega)=-i\hbar k_0\coth\left(\frac{\beta\omega\hbar}{2}\right)\epsilon_{ijl}\frac{\partial}{\partial x_l}\frac{\sin(k_0|\vec{r}-\vec{r}'|)}{|\vec{r}-\vec{r}'|}. \tag{4.10}$$

Equation (4.9) is the well-known expression for the symmetrized correlation function obtained by Landau and Lifshitz in a different manner.[15] The normally ordered correlation functions can be obtained from (3.13)–(3.16) and (4.7), (4.8):

$$\begin{aligned}\mathcal{E}_{ij}^{(N)}(\vec{r},\vec{r}',\omega)&=\mathcal{H}_{ij}^{(N)}(\vec{r},\vec{r}',\omega)=\eta(-\omega)\hbar[1+\coth(\beta\omega\hbar/2)]\left(k_0^2\delta_{ij}+\frac{\partial^2}{\partial x_i\partial x_j}\right)\frac{\sin(k_0|\vec{r}-\vec{r}'|)}{|\vec{r}-\vec{r}'|},\\ \mathcal{G}_{ij}^{(N)}(\vec{r},\vec{r}',\omega)&=-\tilde{\mathcal{G}}_{ij}^{(N)}(\vec{r},\vec{r}',\omega)=-i\hbar k_0\eta(-\omega)\left[1+\coth\left(\frac{\beta\omega\hbar}{2}\right)\right]\epsilon_{ijl}\frac{\partial}{\partial x_l}\frac{\sin(k_0|\vec{r}-\vec{r}'|)}{|\vec{r}-\vec{r}'|}.\end{aligned} \tag{4.11}$$

In the time domain one has, for example,

$$\begin{aligned}\mathcal{E}_{ij}^{(N)}(\vec{r},\vec{r}',t)&=\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\omega\, e^{-i\omega t}2\eta(-\omega)\hbar(e^{\hbar\beta|\omega|}-1)^{-1}\left(\delta_{ij}\nabla^2-\frac{\partial^2}{\partial x_i\partial x_j}\right)\frac{\sin k_0|\vec{r}-\vec{r}'|}{|\vec{r}-\vec{r}'|}\\ &=\frac{\hbar c}{2\pi i}\left(-\delta_{ij}\nabla^2+\frac{\partial^2}{\partial x_i\partial x_j}\right)\int_0^\infty dk_0\, e^{ik_0ct}\sum_{n=1}^{\infty}e^{-\beta\hbar ck_0n}\frac{(e^{ik_0R}-e^{-ik_0R})}{R}\\ &=\frac{\hbar c}{\pi}\left(\frac{\partial^2}{\partial x_i\partial x_j}-\delta_{ij}\nabla^2\right)\sum_{n=1}^{\infty}[(\beta\hbar cn-ict)^2+R^2]^{-1}\\ &=\frac{4\hbar c}{\pi}\sum_{n=1}^{\infty}\left(\frac{\delta_{ij}}{[(\beta\hbar cn-ict)^2+R^2]^2}+\frac{2(R_iR_j-R^2\delta_{ij})}{[(\beta\hbar cn-ict)^2+R^2]^3}\right)\\ &=\mathcal{H}_{ij}^{(N)}(\vec{r},\vec{r}',t),\quad \vec{R}=\vec{r}-\vec{r}'.\end{aligned} \tag{4.12}$$

Similarly the mixed correlation is found to be

$$\begin{aligned}\mathcal{G}_{ij}^{(N)}(\vec{r},\vec{r}',t)&=-\tilde{\mathcal{G}}_{ij}^{(N)}(\vec{r},\vec{r}',t)=\frac{\hbar c}{\pi}\frac{\partial^2}{\partial(ct)\partial x_l}\epsilon_{ijl}\sum_{n=1}^{\infty}[(\beta\hbar cn-ict)^2+R^2]^{-1}\\ &=\frac{-8\hbar ci}{\pi}\epsilon_{ijl}R_l\sum_{n=1}^{\infty}\frac{\beta\hbar cn-ict}{[(\beta\hbar cn-ict)^2+R^2]^3}.\end{aligned} \tag{4.13}$$

The results (4.12) and (4.13) are equivalent to the results of Mehta and Wolf.[3] It is remarkable that we have not used any particular mode expansion of the field operators in deriving (4.12) and (4.13). Antinormally ordered correlations are similarly obtained from (4.5), (4.6), and (3.22)–(3.25).

V. RESPONSE FUNCTIONS FOR THE ELECTROMAGNETIC FIELDS BETWEEN TWO ISOTROPIC, NONMAGNETIC, AND SPATIALLY NONDISPERSIVE BODIES

In this section we will calculate response functions for the case of two identical isotropic, nonmagnetic, and spatially nondispersive bodies separated by vacuum. We assume that each body is characterized by the dielectric function $\epsilon(\omega)$. We moreover assume that the dielectrics occupy volumes $0 \le z \le \infty$ and $-\infty \le z \le -d$, d being the separation between the two bodies. The response functions which we obtain in this section will be very basic in all the applications which we consider in this series of papers.

If we assume that there is an external polarization probe located at $\vec{r} = \vec{r}_0$ in the region $-d \le z \le 0$, then the response of the electric field (magnetic field) would yield the coherence function $\mathcal{E}_{ij}$ ($\mathcal{G}_{ij}$). Similarly the response of the magnetic field (electric field) to an applied magnetization would yield the correlation function $\mathcal{H}_{ij}$ ($\mathcal{G}_{ij}$). We first calculate the response of the electric field to an applied polarization. We must now solve the following set of equations

$$\left.\begin{aligned} &\nabla^2\vec{E} + k_0^2\vec{E} = -4\pi[k_0^2\vec{\mathcal{P}} + \nabla(\nabla\cdot\vec{\mathcal{P}})] \\ &\vec{H} = \nabla\times\vec{E}/ik_0 \end{aligned}\right\} -d \le z \le 0, \tag{5.1}$$

$$\nabla^2\vec{E} + k_0^2\epsilon\vec{E} = 0, \quad \nabla\cdot\vec{E} = 0, \tag{5.2}$$

$$\vec{H} = \nabla\times\vec{E}/ik_0, \quad \text{for } -\infty \le z \le -d, \ 0 \le z \le \infty, \tag{5.3}$$

subject to the appropriate Maxwell boundary conditions at $z = -d$ and $z = 0$. The solution for $\vec{E}$ in the region $-\infty \le z \le -d$ (region III) and $0 \le z \le \infty$ (region I) can be expressed as angular spectrum of plane waves[16]

$$\vec{E}^{(1)}(\vec{r},\omega) = \int\int \vec{\mathcal{E}}^{(1)}(u,v,\omega)e^{i\vec{K}\cdot\vec{r}}\,du\,dv, \quad \vec{K}\cdot\vec{\mathcal{E}}^{(1)} = 0,$$

$$\vec{K} = (u,v,w), \quad w^2 = k_0^2\epsilon - u^2 - v^2, \tag{5.4}$$

$$\vec{E}^{(3)}(\vec{r},\omega) = \int\int \vec{\mathcal{E}}^{(3)}(u,v,\omega)e^{i\vec{K}'\cdot\vec{r}}\,du\,dv,$$

$$\vec{K}'\cdot\vec{\mathcal{E}}^{(3)} = 0, \quad K' = (u,v,-w), \tag{5.5}$$

where the square root is chosen such that $\mathrm{Im}\,w \ge 0$. The solution in the region II ($-d \le z \le 0$) consists of the solution of the homogeneous equation and a particular solution:

$$\vec{E}^{(2)}(\vec{r},\omega) = \int\int [\vec{\mathcal{E}}^{(+)}(u,v,\omega)e^{i\vec{K}_0\cdot\vec{r}} + \vec{\mathcal{E}}^{(-)}(u,v,\omega)e^{i\vec{K}_0'\cdot\vec{r}}]\,du\,dv + \vec{E}_p,$$

$$\vec{K}_0\cdot\vec{\mathcal{E}}^{(+)} = 0, \quad \vec{K}_0'\cdot\vec{\mathcal{E}}^{(-)} = 0, \quad \vec{K}_0 = (u,v,w_0),$$

$$\vec{K}_0' = (u,v,-w_0), \quad w_0^2 = k_0^2 - u^2 - v^2, \tag{5.6}$$

and where $\vec{E}_p$ is given by (4.5) with $\vec{m} = 0$. On using the Weyl representations for Green's function[16]

$$G_0(|\vec{r}-\vec{r}'|) = \frac{i}{2\pi}\int\int \frac{du\,dv}{w_0}\exp[iu(x-x') + iv(y-y') + iw_0|z-z'|], \tag{5.7}$$

$\vec{E}_p$ becomes

$$\vec{E}_p(\vec{r},\omega) = \frac{i}{2\pi}\int\int \frac{du\,dv}{w_0}[k_0^2\vec{p} + \vec{\nabla}(\vec{p}\cdot\vec{\nabla})] \times\exp[iu(x-x_0) + iv(y-y_0) + iw_0|z-z_0|]. \tag{5.8}$$

The angular spectrum representation for the magnetic field is obtained by using the relation $\vec{H} = \vec{\nabla}\times\vec{E}/ik_0$.

On applying Maxwell boundary conditions at $z = 0$, we obtain

$$\vec{\mathcal{E}}_{\parallel}^{(1)} = \vec{\mathcal{E}}_{\parallel}^{(+)} + \vec{\mathcal{E}}_{\parallel}^{(-)} - \nu(\vec{K}_0\times\vec{K}_0\times\vec{p})_{\parallel},$$

$$\nu = (i/2\pi w_0)e^{-i\vec{K}_0\cdot\vec{r}_0}, \tag{5.9}$$

$$\vec{K}\times\vec{\mathcal{E}}^{(1)} = \vec{K}_0\times\vec{\mathcal{E}}^{(+)} + \vec{K}_0'\times\vec{\mathcal{E}}^{(-)} + \nu k_0^2(\vec{K}_0\times\vec{p}). \tag{5.10}$$

Using the transversality of $\vec{\mathcal{E}}^{(1)}$, $\vec{\mathcal{E}}^{(\pm)}$ and on taking the cross product with $\vec{K}$, (5.10) becomes

$$\begin{aligned} k^2\vec{\mathcal{E}}^{(1)} = {} & \vec{\mathcal{E}}^{(+)}(\vec{K}\cdot\vec{K}_0) + \vec{\mathcal{E}}^{(-)}(\vec{K}\cdot\vec{K}_0') - \nu k_0^2(\vec{K}\times\vec{K}_0\times\vec{p}) \\ & - \vec{K}_0(\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}^{(+)})(1 - w/w_0) \\ & - \vec{K}_0'(\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}^{(-)})(1 + w/w_0). \end{aligned}$$

The component of this equation parallel to the surface $z = 0$ is

$$\begin{aligned} & \vec{\mathcal{E}}_{\parallel}^{(+)}[k^2 - (\vec{K}\cdot\vec{K}_0)] + \vec{\mathcal{E}}_{\parallel}^{(-)}[k^2 - (\vec{K}\cdot\vec{K}_0')] \\ & + \vec{K}_{\parallel}(\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}^{(+)})(1 - w/w_0) + \vec{K}_{\parallel}(\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}^{(-)})(1 + w/w_0) \\ & = \nu[k^2(\vec{K}_0\times\vec{K}_0\times\vec{p})_{\parallel} - k_0^2(\vec{K}\times\vec{K}_0\times\vec{p})_{\parallel}], \end{aligned} \tag{5.11}$$

where (5.9) has been used to eliminate $\vec{\mathcal{E}}_{\parallel}^{(1)}$. From (5.11) one easily obtains the following two equations:

$$\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}_{\parallel}^{(+)} + \frac{w_0\epsilon + w}{w_0\epsilon - w}\,\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}_{\parallel}^{(-)} = \nu w_0[p_{\perp}k_{\parallel}^2 - w_0(\vec{K}_{\parallel}\cdot\vec{p}_{\parallel})], \tag{5.12}$$

$$\vec{K}_{\parallel}\times\vec{\mathcal{E}}_{\parallel}^{(+)} + \frac{w + w_0}{w - w_0}\vec{K}_{\parallel}\times\vec{\mathcal{E}}_{\parallel}^{(-)} = -\nu k_0^2(\vec{K}_{\parallel}\times\vec{p}_{\parallel}), \tag{5.13}$$

where $\vec{K}_{\parallel}$ is the vector parallel to the surface $z = 0$ and has components $\vec{K}_{\parallel} = (u,v,0)$ and $p_{\perp}$ is the component of $\vec{p}$ along z direction.

Applying now the Maxwell boundary conditions at $z = -d$, we obtain

$$\vec{\mathcal{E}}_{\parallel}^{(3)}e^{iwd} = \vec{\mathcal{E}}_{\parallel}^{(+)}e^{-iw_0d} + \vec{\mathcal{E}}_{\parallel}^{(-)}e^{iw_0d} - \nu'(\vec{K}_0'\times\vec{K}_0'\times\vec{p})_{\parallel}, \tag{5.14}$$

$$(\vec{K}'\times\vec{\mathcal{E}}^{(3)})e^{iw_0d} = (\vec{K}_0\times\vec{\mathcal{E}}^{(+)})e^{-iw_0d} + (\vec{K}_0'\times\vec{\mathcal{E}}^{(-)})e^{iw_0d} + \nu' k_0^2\vec{K}_0'\times\vec{p},$$

$$\nu' = (i/2\pi w_0)e^{-i\vec{K}_0'\cdot\vec{r}_0 + iw_0d}. \tag{5.15}$$

Carrying out an analysis similar to that which led to (5.12) and (5.13), we find that (5.14) and (5.15) lead to

$$(\vec{K}_{\parallel}\times\vec{\mathcal{E}}_{\parallel}^{(+)})e^{-iw_0 d}+\frac{w-w_0}{w+w_0}(\vec{K}_{\parallel}\times\vec{\mathcal{E}}_{\parallel}^{(-)})e^{iw_0 d} = -\nu' k_0^2(\vec{K}_{\parallel}\times\vec{p}_{\parallel})\frac{w-w_0}{w+w_0}, \quad (5.16)$$

$$(\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}_{\parallel}^{(+)})e^{-iw_0 d}+(\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}_{\parallel}^{(-)})e^{iw_0 d}\frac{w_0\epsilon-w}{w_0\epsilon+w} = -\nu' w_0\frac{w_0\epsilon-w}{w_0\epsilon+w}[k_{\parallel}^2 p_{\perp}+w_0(\vec{K}_{\parallel}\cdot\vec{p}_{\parallel})]. \quad (5.17)$$

On solving (5.12), (5.13), (5.16) and (5.17) for the four unknown $(\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}_{\parallel}^{(\pm)})$, $(\vec{K}_{\parallel}\times\vec{\mathcal{E}}_{\parallel}^{(\pm)})$, we obtain

$$\vec{K}_{\parallel}\times\vec{\mathcal{E}}_{\parallel}^{(-)}=\frac{ik_0^2}{2\pi w_0}D_2^{-1}(\vec{K}_{\parallel}\times\vec{p}_{\parallel})\left(\frac{w+w_0}{w-w_0}e^{-i\vec{K}_0\cdot\vec{r}_0-2iw_0 d}-e^{-i\vec{K}_0'\cdot\vec{r}_0}\right), \quad (5.18)$$

$$\vec{K}_{\parallel}\times\vec{\mathcal{E}}_{\parallel}^{(+)}=\frac{-ik_0^2}{2\pi w_0}D_2^{-1}(\vec{K}_{\parallel}\times\vec{p}_{\parallel})\left(e^{-i\vec{K}_0\cdot\vec{r}_0}-\frac{w+w_0}{w-w_0}e^{-i\vec{K}_0'\cdot\vec{r}_0}\right), \quad (5.19)$$

$$\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}_{\parallel}^{(-)}=\frac{-iw_0}{2\pi w_0}D_1^{-1}\left[(k_{\parallel}^2 p_{\perp}+w_0\vec{K}_{\parallel}\cdot\vec{p}_{\parallel})e^{-i\vec{K}_0'\cdot\vec{r}_0}+\frac{w_0\epsilon+w}{w_0\epsilon-w}(k_{\parallel}^2 p_{\perp}-w_0\vec{K}_{\parallel}\cdot\vec{p}_{\parallel})e^{-i\vec{K}_0\cdot\vec{r}_0-2iw_0 d}\right]=w_0\mathcal{E}_z^{(-)}, \quad (5.20)$$

$$\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}_{\parallel}^{(+)}=\frac{iw_0}{2\pi w_0}D_1^{-1}\left[(k_{\parallel}^2 p_{\perp}-w_0\vec{K}_{\parallel}\cdot\vec{p}_{\parallel})e^{-i\vec{K}_0\cdot\vec{r}_0}+\frac{w_0\epsilon+w}{w_0\epsilon-w}(k_{\parallel}^2 p_{\perp}+w_0\vec{K}_{\parallel}\cdot\vec{p}_{\parallel})e^{-i\vec{K}_0'\cdot\vec{r}_0}\right]=-w_0\mathcal{E}_z^{(+)}, \quad (5.21)$$

where D_1 and D_2 are given by

$$D_1=1-\left(\frac{w_0\epsilon+w}{w_0\epsilon-w}\right)^2 e^{-2iw_0 d},\quad D_2=1-\left(\frac{w+w_0}{w-w_0}\right)^2 e^{-2iw_0 d}. \quad (5.22)$$

The tangential components of $\vec{\mathcal{E}}$ are given by

$$\mathcal{E}_x=\frac{u(\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}_{\parallel})-v(\vec{K}_{\parallel}\times\vec{\mathcal{E}}_{\parallel})}{k_{\parallel}^2},\quad \mathcal{E}_y=\frac{u(\vec{K}_{\parallel}\times\vec{\mathcal{E}}_{\parallel})+v(\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}_{\parallel})}{k_{\parallel}^2}. \quad (5.23)$$

The complete electric field is obtained by substituting (5.18)–(5.23) into (5.6). The $\vec{E}_p$ term leads to the usual translationally invariant response functions which we have computed in Sec. IV. We have, for example, the response of the z component of the electric field to an applied $p_{\perp}$:

$$\chi_{zzEE}^{(1)}(\vec{r},\vec{r}_0,\omega)=\frac{-i}{2\pi}\int\int\frac{du\,dv}{w_0}D_1^{-1}(u^2+v^2)\left[e^{i\vec{K}_0'\cdot(\vec{r}-\vec{r}_0)}+e^{i\vec{K}_0\cdot(\vec{r}-\vec{r}_0)}+\frac{w_0\epsilon+w}{w_0\epsilon-w}\times(e^{i\vec{K}_0\cdot\vec{r}-i\vec{K}_0'\cdot\vec{r}_0}+e^{i\vec{K}_0'\cdot\vec{r}-i\vec{K}_0\cdot\vec{r}_0-2iw_0 d})\right], \quad (5.24)$$

where we have ignored the usual translationally invariant contribution.[17] As mentioned earlier the response function χ_{ijHE} can be obtained from $\vec{H}=\vec{\nabla}\times\vec{E}/ik_0$ and the relations (5.6), (5.18)–(5.23).

We now show how to calculate the response function $\chi_{ijHH}(\vec{r},\vec{r}_0,\omega)$. We have seen that in free space $\chi_{ijEE}=\chi_{ijHH}$; however, for a bounded medium it does not hold as the boundary conditions satisfied by $\vec{E}$ and $\vec{H}$ are asymmetrical for a dielectric medium. To calculate χ_{ijHH} we apply a magnetization at $\vec{r}=\vec{r}_0$. The equations to be solved are now

$$\left.\begin{aligned}&(\nabla^2+k_0^2)\vec{H}=-4\pi[k_0^2\vec{\mathfrak{M}}+\nabla(\nabla\cdot\vec{\mathfrak{M}})]\\&\vec{E}=-\vec{\nabla}\times\vec{H}/ik_0\end{aligned}\right\}-d\le z\le 0, \quad (5.25)$$

$$(\nabla^2+k_0^2\epsilon)\vec{H}=0,\quad \vec{E}=-\vec{\nabla}\times\vec{H}/ik_0\epsilon,\quad \vec{\nabla}\cdot\vec{H}=0,\quad \text{for } -\infty\le z\le -d,\ 0\le z\le\infty. \quad (5.26)$$

On using continuity of $\vec{H}$ and tangential $\vec{E}$ and on expressing each of the fields in the angular spectrum representation, we obtain on carrying out an analysis very similar to that which led to (5.18)–(5.23):

$$\vec{K}_{\parallel}\cdot\vec{\mathcal{H}}_{\parallel}^{(-)}=\frac{iw_0}{2\pi w_0}D_2^{-1}\left(\frac{w+w_0}{w-w_0}(k_{\parallel}^2 m_{\perp}-w_0\vec{K}_{\parallel}\cdot\vec{m}_{\parallel})e^{-i\vec{K}_0\cdot\vec{r}_0}e^{-2iw_0 d}-(k_{\parallel}^2 m_{\perp}+w_0\vec{K}_{\parallel}\cdot\vec{m}_{\parallel})e^{-i\vec{K}_0'\cdot\vec{r}_0}\right)=w_0\mathcal{H}_z^{(-)}, \quad (5.27)$$

$$\vec{K}_{\parallel}\cdot\vec{\mathcal{H}}_{\parallel}^{(+)}=\frac{iw_0}{2\pi w_0}D_2^{-1}\left((k_{\parallel}^2 m_{\perp}-w_0\vec{K}_{\parallel}\cdot\vec{m}_{\parallel})e^{-i\vec{K}_0\cdot\vec{r}_0}-\frac{w+w_0}{w-w_0}(k_{\parallel}^2 m_{\perp}+w_0\vec{K}_{\parallel}\cdot\vec{m}_{\parallel})e^{-i\vec{K}_0'\cdot\vec{r}_0}\right)=-w_0\mathcal{H}_z^{(+)}, \quad (5.28)$$

$$\vec{K}_{\parallel}\times\vec{\mathcal{H}}_{\parallel}^{(-)}=\frac{-ik_0^2}{2\pi w_0}D_1^{-1}(\vec{K}_{\parallel}\times\vec{m}_{\parallel})\left(e^{-i\vec{K}_0'\cdot\vec{r}_0}+\frac{w_0\epsilon+w}{w_0\epsilon-w}e^{-i\vec{K}_0\cdot\vec{r}_0-2iw_0 d}\right), \quad (5.29)$$

$$\vec{K}_{\parallel}\times\vec{\mathcal{H}}_{\parallel}^{(+)}=\frac{-ik_0^2}{2\pi w_0}D_1^{-1}(\vec{K}_{\parallel}\times\vec{m}_{\parallel})\left(e^{-i\vec{K}_0\cdot\vec{r}_0}+\frac{w_0\epsilon+w}{w_0\epsilon-w}e^{-i\vec{K}_0'\cdot\vec{r}_0}\right),\tag{5.30}$$

$$\mathcal{H}_x=\frac{u(\vec{K}_{\parallel}\cdot\vec{\mathcal{H}}_{\parallel})-v(\vec{K}_{\parallel}\times\vec{\mathcal{H}}_{\parallel})}{k_{\parallel}^2},\quad \mathcal{H}_y=\frac{u(\vec{K}_{\parallel}\times\vec{\mathcal{H}}_{\parallel})+v(\vec{K}_{\parallel}\cdot\vec{\mathcal{H}}_{\parallel})}{k_{\parallel}^2}.\tag{5.31}$$

This completes our calculation of all the response functions. From (5.27)–(5.31) we have, for example,

$$\chi_{zzHH}^{(1)}(\vec{r},\vec{r}_0,\omega)=\frac{-i}{2\pi}\int\int\frac{du\,dv}{w_0}D_2^{-1}(u^2+v^2)\bigg(e^{i\vec{K}_0\cdot(\vec{r}-\vec{r}_0)}+e^{i\vec{K}_0'\cdot(\vec{r}-\vec{r}_0)}-\frac{w+w_0}{w-w_0}\times(e^{-i\vec{K}_0'\cdot\vec{r}_0+i\vec{K}_0\cdot\vec{r}}+e^{-i\vec{K}_0\cdot\vec{r}_0+i\vec{K}_0'\cdot\vec{r}-2iw_0d})\bigg),\tag{5.32}$$

where as before we have written here only the surface-dependent contribution. We next discuss some special cases.

A. Each dielectric replaced by a perfect conductor

On taking the limit of a perfect conductor, Eqs. (5.18)–(5.22) reduce to

$$\vec{K}_{\parallel}\times\vec{\mathcal{E}}_{\parallel}^{(-)}=\frac{ik_0^2}{2\pi w_0}D_0^{-1}(\vec{K}_{\parallel}\times\vec{p}_{\parallel})(e^{-2iw_0d-i\vec{K}_0\cdot\vec{r}_0}-e^{-i\vec{K}_0'\cdot\vec{r}_0}),\tag{5.33}$$

$$\vec{K}_{\parallel}\times\vec{\mathcal{E}}_{\parallel}^{(+)}=\frac{-ik_0^2}{2\pi w_0}D_0^{-1}(\vec{K}_{\parallel}\times\vec{p}_{\parallel})(e^{-i\vec{K}_0\cdot\vec{r}_0}-e^{-i\vec{K}_0'\cdot\vec{r}_0}),\tag{5.34}$$

$$\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}_{\parallel}^{(-)}=\frac{-iw_0}{2\pi w_0}D_0^{-1}[(k_{\parallel}^2p_{\perp}+w_0\vec{K}_{\parallel}\cdot\vec{p}_{\parallel})e^{-i\vec{K}_0'\cdot\vec{r}_0}+(k_{\parallel}^2p_{\perp}-w_0\vec{K}_{\parallel}\cdot\vec{p}_{\parallel})e^{-i\vec{K}_0\cdot\vec{r}_0-2iw_0d}]=w_0\mathcal{E}_z^{(-)},\tag{5.35}$$

$$\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}_{\parallel}^{(+)}=\frac{+iw_0}{2\pi w_0}D_0^{-1}[(k_{\parallel}^2p_{\perp}+w_0\vec{K}_{\parallel}\cdot\vec{p}_{\parallel})e^{-i\vec{K}_0'\cdot\vec{r}_0}+(k_{\parallel}^2p_{\perp}-w_0\vec{K}_{\parallel}\cdot\vec{p}_{\parallel})e^{-i\vec{K}_0\cdot\vec{r}_0}]=-w_0\mathcal{E}_z^{(+)},\tag{5.36}$$

$$D_0=1-e^{-2iw_0d},\tag{5.37}$$

and the corresponding magnetic field equations reduce to

$$\vec{K}_{\parallel}\cdot\vec{\mathcal{H}}_{\parallel}^{(-)}=\frac{iw_0}{2\pi w_0}D_0^{-1}[(k_{\parallel}^2m_{\perp}-w_0\vec{K}_{\parallel}\cdot\vec{m}_{\parallel})e^{-i\vec{K}_0\cdot\vec{r}_0-2iw_0d}-(k_{\parallel}^2m_{\perp}+w_0\vec{K}_{\parallel}\cdot\vec{m}_{\parallel})e^{-i\vec{K}_0'\cdot\vec{r}_0}]=w_0\mathcal{H}_z^{(-)},\tag{5.38}$$

$$\vec{K}_{\parallel}\cdot\vec{\mathcal{H}}_{\parallel}^{(+)}=\frac{iw_0}{2\pi w_0}D_0^{-1}[(k_{\parallel}^2m_{\perp}-w_0\vec{K}_{\parallel}\cdot\vec{m}_{\parallel})e^{-i\vec{K}_0\cdot\vec{r}_0}-(k_{\parallel}^2m_{\perp}+w_0\vec{K}_{\parallel}\cdot\vec{m}_{\parallel})e^{-i\vec{K}_0'\cdot\vec{r}_0}]=-w_0\mathcal{H}_z^{(+)},\tag{5.39}$$

$$\vec{K}_{\parallel}\times\vec{\mathcal{H}}_{\parallel}^{(-)}=\frac{-ik_0^2}{2\pi w_0}D_0^{-1}(\vec{K}_{\parallel}\times\vec{m}_{\parallel})\times(e^{-i\vec{K}_0'\cdot\vec{r}_0}+e^{-i\vec{K}_0\cdot\vec{r}_0-2iw_0d}),\tag{5.40}$$

$$\vec{K}_{\parallel}\times\vec{\mathcal{H}}_{\parallel}^{(+)}=\frac{-ik_0^2}{2\pi w_0}D_0^{-1}(\vec{K}_{\parallel}\times\vec{m}_{\parallel})\times(e^{-i\vec{K}_0\cdot\vec{r}_0}+e^{-i\vec{K}_0'\cdot\vec{r}_0}).\tag{5.41}$$

B. Dielectric occupying $-\infty\leqslant z\leqslant -d$ absent

On taking the limit $d\to\infty$, Eqs. (5.18)–(5.22) and (5.27)–(5.30) reduce to

$$\vec{\mathcal{E}}^{(+)}=\vec{\mathcal{H}}^{(+)}=0,\tag{5.42}$$

$$\vec{K}_{\parallel}\times\vec{\mathcal{E}}_{\parallel}^{(-)}=\frac{-ik_0^2}{2\pi w_0}\vec{K}_{\parallel}\times\vec{p}_{\parallel}\frac{w-w_0}{w+w_0}e^{-i\vec{K}_0\cdot\vec{r}_0},\tag{5.43}$$

$$\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}_{\parallel}^{(-)}=\frac{iw_0}{2\pi w_0}(k_{\parallel}^2p_{\perp}-w_0\vec{K}_{\parallel}\cdot\vec{p}_{\parallel})\times\frac{w_0\epsilon-w}{w_0\epsilon+w}e^{-i\vec{K}_0\cdot\vec{r}_0}=w_0\mathcal{E}_z^{(-)},\tag{5.44}$$

$$\vec{K}_{\parallel}\times\vec{\mathcal{H}}_{\parallel}^{(-)}=\frac{ik_0^2}{2\pi w_0}\vec{K}_{\parallel}\times\vec{m}_{\parallel}\frac{w_0\epsilon-w}{w_0\epsilon+w}e^{-i\vec{K}_0\cdot\vec{r}_0},\tag{5.45}$$

$$\vec{K}_{\parallel}\cdot\vec{\mathcal{H}}_{\parallel}^{(-)}=\frac{-iw_0}{2\pi w_0}(k_{\parallel}^2m_{\perp}-w_0\vec{K}_{\parallel}\cdot\vec{m}_{\parallel})\times\frac{w-w_0}{w+w_0}e^{-i\vec{K}_0\cdot\vec{r}_0}=w_0\mathcal{H}_z^{(-)}.\tag{5.46}$$

In the limit of infinite conductivity (perfect conductor), these equations further reduce to

$$\vec{K}_{\parallel}\times\vec{\mathcal{E}}_{\parallel}^{(-)}=\frac{-ik_0^2}{2\pi w_0}(\vec{K}_{\parallel}\times\vec{p}_{\parallel})e^{-i\vec{K}_0\cdot\vec{r}_0},\tag{5.47}$$

$$\vec{K}_{\parallel}\cdot\vec{\mathcal{E}}_{\parallel}^{(-)}=\frac{iw_0}{2\pi w_0}(k_{\parallel}^2p_{\perp}-w_0\vec{K}_{\parallel}\cdot\vec{p}_{\parallel})e^{-i\vec{K}_0\cdot\vec{r}_0},\tag{5.48}$$

$$\vec{K}_{\parallel}\times\vec{\mathcal{H}}_{\parallel}^{(-)}=\frac{ik_0^2}{2\pi w_0}(\vec{K}_{\parallel}\times\vec{m}_{\parallel})e^{-i\vec{K}_0\cdot\vec{r}_0},\tag{5.49}$$

$$\vec{K}_{\parallel}\cdot\vec{\mathcal{H}}_{\parallel}^{(-)}=\frac{-iw_0}{2\pi w_0}(k_{\parallel}^2m_{\perp}-w_0\vec{K}_{\parallel}\cdot\vec{m}_{\parallel})e^{-i\vec{K}_0\cdot\vec{r}_0}.\tag{5.50}$$

VI. RESPONSE FUNCTIONS FOR THE ELECTROMAGNETIC FIELDS BETWEEN A SPATIALLY DISPERSIVE AND SPATIALLY NONDISPERSIVE DIELECTRIC

We now consider electromagnetic field fluctuations in the domain bounded by a spatially dispersive and a spatially nondispersive dielectric. Such a calculation will be useful in our treatment of dispersion force between a spatially dispersive and a spatially nondispersive dielectric.[18] We treat the spatially dispersive body[19] in the effective-mass approximation and for simplicity only consider the case of bodies separated by small distances, i.e., when the retardation effects can be ignored.

We write the dielectric function, in the effective-mass approximation, as

$$\epsilon(\vec{k},\omega)=\epsilon_0+\chi/(k^2-\mu^2), \tag{6.1}$$

where

$$\chi=4\pi\alpha m^*\omega_0/\hbar, \qquad \mu^2=(m^*/\hbar\omega_0)(\omega^2-\omega_0^2+i\omega\Gamma), \tag{6.2}$$

and where m^* is the effective mass, Γ is a phenomenological damping, and α is related to the oscillator strength. It should be noted that an electron gas in the hydrodynamic approximation is described by a similar dielectric function. We must know the structure of the electromagnetic field inside the spatially dispersive medium. For the model (6.1) we will obtain the structure of electromagnetic field in the approximation that electric induction $\vec{D}$ and $\vec{E}$ are related by[20-22]

$$\vec{D}(\vec{r},\omega)=\epsilon_0\vec{E}(\vec{r},\omega)+(\chi/4\pi)\int_V G_\mu(|\vec{r}-\vec{r}\,'|)\times\vec{E}(\vec{r}\,',\omega)\,d^3r', \tag{6.3}$$

where the integration is over the volume of the medium only and G_μ is the Green's function defined by

$$G_\mu(|\vec{r}-\vec{r}\,'|)=e^{i\mu|\vec{r}-\vec{r}\,'|}/|\vec{r}-\vec{r}\,'|. \tag{6.4}$$

The scalar potential Φ satisfies the equation

$$\nabla^2\Phi+(\chi/4\pi\epsilon_0)\nabla\cdot\int_V G_\mu(|\vec{r}-\vec{r}\,'|)\,\nabla'\Phi(\vec{r}\,',\omega)\,d^3r'=0. \tag{6.5}$$

Let us assume that the spatially dispersive medium occupies the domain $0\le z\le\infty$. Then it can be shown that the solution of (6.5) is given by

$$\Phi(\vec{r},\omega)=\int\int du\,dv\sum\Phi_j e^{iux+ivy+iw_jz}, \tag{6.6}$$

where

$$w_1^2=-(u^2+v^2),\quad w_2^2=k_2^2-(u^2+v^2),\quad k_2^2=\mu^2-\chi/\epsilon_0. \tag{6.7}$$

The amplitudes Φ_1 and Φ_2 are not independent but are related linearly by[23]

$$\sum w_j\Phi_j=k_2^2(w_2-w_\mu)^{-1}\Phi_2, \qquad w_\mu^2=\mu^2-(u^2+v^2), \tag{6.8}$$

which we will write as

$$\Phi_2=-\alpha\Phi_1. \tag{6.9}$$

It can be further shown that the normal component of electric induction at $z=0$ is given by

$$D_z(x,y,0,\omega)=\int\int du\,dv\,e^{iux+ivy}\sum_j(-i\,\Phi_j\beta_j), \tag{6.10}$$

where

$$\beta_j=w_j\epsilon_j-(\chi w_1^2/2w_\mu^2)(w_j-w_\mu)^{-1}, \qquad \epsilon_j=\epsilon_0+\chi/(w_j^2-w_\mu^2). \tag{6.11}$$

Equations (6.6)–(6.11) completely characterize the electromagnetic fields inside the spatially dispersive medium. Having obtained the fields inside, we proceed to calculate the response of the potential to an applied charge at the point $\vec{r}_0$, i.e., $\rho(\vec{r},\omega)=\rho(\omega)\delta(\vec{r}-\vec{r}_0)$. The perturbing Hamiltonian has the form

$$H_{\text{ext}}=\int\rho(\vec{r},t)\Phi(\vec{r},t)\,d^3r. \tag{6.12}$$

It is now clear that the response function

$$\chi(\vec{r},\vec{r}\,',\omega)=-\delta\langle\Phi(r,\omega)\rangle/\delta\rho(\vec{r}\,',\omega), \tag{6.13}$$

will yield the fluctuation correlation of the scalar potential at two different space-time points. We consider the following situation: the dielectric characterized by $\epsilon_3(\omega)$ occupies the domain $-\infty\le z\le -d$ (region III) and the spatially dispersive dielectric occupies the volume $0\le z\le\infty$ (region I). We assume that the external charge is located at $\vec{r}_0$ in the vacuum (the region II, $-d\le z\le 0$). In the region II we have Poisson's equation

$$\nabla^2\Phi=-4\pi\rho\delta(\vec{r}-\vec{r}_0), \tag{6.14}$$

and in region III, the Laplace equation

$$\nabla^2\Phi=0. \tag{6.15}$$

We write the solution in the region II, III in the form of angular spectrum of plane waves

$$\Phi^{(3)}(\vec{r},\omega)=\int\int du\,dv\,\Phi^{(3)}(u,v,\omega)e^{iux+ivy-iw_1z}, \tag{6.16}$$

$$\Phi^{(2)}(\vec{r},\omega)=\int\int du\,dv\,[\Phi^{(+)}(u,v,\omega)e^{iw_1z}+\Phi^{(-)}(u,v,\omega)\times e^{-iw_1z}+(i\rho/2\pi w_1)\times e^{-iux_0-ivy_0+iw_1|z-z_0|}]\,e^{iux+ivy}, \tag{6.17}$$

where the Weyl representation of the Green's function,

$$\frac{1}{|\vec{r}-\vec{r}_0|}=\frac{i}{2\pi}\int\int\frac{du\,dv}{w_1}\exp[iu(x-x_0)+iv(y-y_0)+iw_1|z-z_0|], \tag{6.18}$$

has been used. To calculate $\Phi^{(\pm)}$, $\Phi^{(3)}$ we use the continuity of Φ and the normal component of the electric induction. Applying boundary conditions at $z=0$ we have

$$\Phi_1+\Phi_2=\Phi^{(+)}+\Phi^{(-)}+\frac{i\rho}{2\pi w_1}e^{-i\vec{K}\cdot\vec{r}_0},$$

$$\vec{K}=(u,v,w_1), \tag{6.19}$$

$$\sum_j\beta_j\Phi_j=w_1[\Phi^{(+)}-\Phi^{(-)}+(i\rho/2\pi w_1)e^{-i\vec{K}\cdot\vec{r}_0}]. \tag{6.20}$$

On using (6.9), (6.19), and (6.20) we find the relation

$$\Phi^{(+)}-\Phi^{(-)}\frac{w_1(1-\alpha)+\beta_1-\beta_2\alpha}{w_1(1-\alpha)-\beta_1+\beta_2\alpha}=\frac{-i\rho}{2\pi w_1}e^{-i\vec{K}\cdot\vec{r}_0}. \tag{6.21}$$

Similarly on applying boundary conditions at $z=-d$ we obtain

$$\Phi^{(+)}+\frac{\epsilon_3-1}{\epsilon_3+1}\left(\Phi^{(-)}e^{2iw_1d}+\frac{i\rho}{2\pi w_1}e^{2iw_1d-i\vec{K}'\cdot\vec{r}_0}\right)=0$$

$$\vec{K}'=(u,v,-w_1). \tag{6.22}$$

On solving (6.21) and (6.22) we obtain

$$\Phi^{(-)}=\rho D^{-1}\frac{i}{2\pi w_1}\left(\frac{\epsilon_3+1}{\epsilon_3-1}e^{-2iw_1d-i\vec{K}\cdot\vec{r}_0}-e^{-i\vec{K}'\cdot\vec{r}_0}\right), \tag{6.23}$$

$$\Phi^{(+)}=\frac{-i\rho}{2\pi w_1}D^{-1}\left(e^{-i\vec{K}\cdot\vec{r}_0}-e^{-i\vec{K}'\cdot\vec{r}_0}\times\frac{\beta_1-\beta_2\alpha+w_1(1-\alpha)}{\beta_1-\beta_2\alpha-w_1(1-\alpha)}\right), \tag{6.24}$$

where

$$D=\left(1-\frac{\epsilon_3+1}{\epsilon_3-1}\,\frac{\beta_1-\beta_2\alpha+w_1(1-\alpha)}{\beta_1-\beta_2\alpha-w_1(1-\alpha)}\,e^{-2iw_1d}\right). \tag{6.25}$$

The response function (6.13) is therefore given by

$$\chi(\vec{r},\vec{r}\,',\omega)=\chi^{(0)}(\vec{r},\vec{r}\,',\omega)+\chi^{(1)}(\vec{r},\vec{r}\,',\omega),\qquad -d\leq z,z'\leq 0, \tag{6.26}$$

where $\chi^{(0)}$ is the free-space value

$$\chi^{(0)}(\vec{r},\vec{r}\,',\omega)=-|\vec{r}-\vec{r}\,'|^{-1}, \tag{6.27}$$

and $\chi^{(1)}$ is equal to

$$\chi^{(1)}(\vec{r},\vec{r}_0,\omega)=\frac{i}{2\pi}\int\int\frac{du\,dv}{w_1D}e^{iu(x-x_0)+iv(y-y_0)}\left(e^{iw_1(z-z_0)}+e^{-iw_1(z-z_0)}-\frac{\epsilon_3+1}{\epsilon_3-1}e^{-iw_1(z+z_0+2d)}-\frac{\beta_1-\beta_2\alpha+w_1(1-\alpha)}{\beta_1-\beta_2\alpha-w_1(1-\alpha)}e^{iw_1(z+z_0)}\right). \tag{6.28}$$

The response function $\chi_{ijEE}(\vec{r},\vec{r}_0,\omega)$ is obtained from $\chi(\vec{r},\vec{r}_0,\omega)$ by using

$$\chi_{ijEE}(\vec{r},\vec{r}_0,\omega)=\frac{\partial^2}{\partial x_i\partial x_{0j}}\chi(\vec{r},\vec{r}_0,\omega). \tag{6.29}$$

The response function (6.29) will be useful in the calculation of among other things the dispersion force between a spatially dispersive body and spatially nondispersive body. The response function (6.28) takes a simpler form if the dielectric ϵ_3 is absent

$$\chi^{(1)}(\vec{r},\vec{r}_0,\omega)=\frac{i}{2\pi}\int\int\frac{du\,dv}{w_1}\,\frac{\beta_1-\beta_2\alpha-w_1(1-\alpha)}{\beta_1-\beta_2\alpha+w_1(1-\alpha)}\exp[iu(x-x_0)+iv(y-y_0)+iw_1(z+z_0)]. \tag{6.30}$$

VII. BLACK-BODY FLUCTUATIONS IN FINITE GEOMETRIES

In this section we consider how the black-body fluctuations are modified due to the presence of boundaries. We have already computed all the relevant response functions in Sec. V and these in conjunction with (3.13)–(3.20) will lead to all the correlation functions $\mathcal{E}_{ij}$, $\mathcal{H}_{ij}$, $\mathcal{G}_{ij}$, $\bar{\mathcal{G}}_{ij}$. We are treating fluctuations in a specialized geometry, i.e., in the region bounded by two dielectrics. One has, for example,

$$\mathcal{E}_{zz}^{(N)}(\vec{r},\vec{r}_0,\omega)=\mathcal{E}_{zz}^{(N)(0)}(\vec{r},\vec{r}_0,\omega)+\mathcal{E}_{zz}^{(N)(1)}(\vec{r},\vec{r}_0,\omega), \tag{7.1}$$

where $\mathcal{E}_{zz}^{(N)(0)}(\vec{r},\vec{r}_0,\omega)$ is the usual contribution given by (4.12) and $\mathcal{E}_{zz}^{(N)(1)}$ is the surface dependent contribution given by

$$\mathcal{E}_{zz}^{(N)(1)}(\vec{r},\vec{r}_0,\omega)=2\hbar\eta(-\omega)(1-e^{-\beta\hbar\omega})^{-1}\times\mathrm{Im}\chi_{zzEE}^{(1)}(\vec{r},\vec{r}_0,\omega), \tag{7.2}$$

where $\chi_{zzEE}^{(1)}(\vec{r},\vec{r}_0,\omega)$ is given by (5.24). In order to study some new features of the black-body radiation in finite geometries, we treat the simplified situation: Consider the fluctuations in the region $-\infty\leq z\leq 0$ (vacuum) bounded by a perfect conductor at one end, $z=0$. In this case formulas (5.47)–(5.50) apply. Equations (5.47) and (5.48) can be rearranged in the form

$$\vec{\mathcal{E}}_{\parallel}^{(-)}=(i/2\pi w_0)e^{-i\vec{K}_0\cdot\vec{r}_0}[\vec{K}_{\parallel}(\vec{K}_{\parallel}\cdot\vec{p})-k_0^2\vec{p}_{\parallel}+p_{\perp}w_0\vec{K}_{\parallel}], \tag{7.3}$$

$$\mathcal{E}_{z}^{(-)}=(i/2\pi w_0)e^{-i\vec{K}_0\cdot\vec{r}_0}[p_{\perp}k_{\parallel}^2-w_0(\vec{K}_{\parallel}\cdot\vec{p}_{\parallel})]. \tag{7.4}$$

To obtain $\chi_{iJHE}^{(1)}$ we calculate the magnetic field from (5.47), (5.48), and the relation $\vec{H}=\nabla\times\vec{E}/ik_0$. It is seen from (7.3) and (7.4) that the contribution $\vec{E}$ from the solution of the homogeneous equation is

$$\vec{E}_{\mathrm{hom}}=[(\vec{p}\cdot\vec{\nabla}_{\perp})\vec{\nabla}-(\vec{p}\cdot\vec{\nabla}_{\parallel})\vec{\nabla}-k_0^2\vec{p}+2k_0^2(\hat{z}\cdot\vec{p})\hat{z}]\,e^{ik_0R'}/R', \tag{7.5}$$

where $\vec{R}'$ is the distance of the point $\vec{r}$ from the image of $\vec{r}_0$, i.e.,

$$R_x'=x-x_0,\quad R_y'=y-y_0,\quad R_z'=z+z_0, \tag{7.6}$$

and hence $\vec{H}_{\mathrm{hom}}$ is equal to

$$\vec{H}_{\mathrm{hom}}=-ik_0\vec{p}_0\times\vec{\nabla}(e^{ik_0R'}/R'),\qquad \vec{p}_{0\parallel}=\vec{p}_{\parallel},\quad p_{0\perp}=-p_{\perp}. \tag{7.7}$$

From (7.5), (7.7), (5.49), (5.50) we obtain the response functions

$$\chi_{ijEE}^{(1)}(\vec{r},\vec{r}_0,\omega)=\left(\frac{\partial^2}{\partial x_i\partial x_j}(2\delta_{j3}-1)-k_0^2\delta_{ij}+2k_0^2\delta_{i3}\delta_{j3}\right)\frac{e^{ik_0R'}}{R'} \tag{7.8}$$

$$=-\chi_{ijHH}^{(1)}(\vec{r},\vec{r}_0,\omega), \tag{7.9}$$

$$\chi_{ijHE}^{(1)}(\vec{r},\vec{r}_0,\omega)=ik_0(2\delta_{j3}-1)\epsilon_{ijl}\frac{\partial}{\partial x_l}\frac{e^{ik_0R'}}{R'} \tag{7.10}$$

$$=+\chi_{ijEH}^{(1)}(\vec{r},\vec{r}_0,\omega). \tag{7.11}$$

The symmetry properties (7.9) and (7.11) are to be compared with those for translationally invariant response functions

$$\chi_{ijEE}^{(0)}(\vec{r},\vec{r}_0,\omega)=\chi_{ijHH}^{(0)}(\vec{r},\vec{r}_0,\omega),\qquad \chi_{ijEH}^{(0)}(\vec{r},\vec{r}_0,\omega)=-\chi_{ijHE}^{(0)}(\vec{r},\vec{r}_0,\omega). \tag{7.12}$$

The correlation functions have similar properties

$$\begin{aligned}\mathcal{E}_{ij}^{(N)(0)}(\vec{r},\vec{r}_0,\omega)&=\mathcal{H}_{ij}^{(N)(0)}(\vec{r},\vec{r}_0,\omega),\\ \mathcal{G}_{ij}^{(N)(0)}(\vec{r},\vec{r}_0,\omega)&=-\tilde{\mathcal{G}}_{ij}^{(N)(0)}(\vec{r},\vec{r}_0,\omega),\\ \mathcal{E}_{ij}^{(N)(1)}(\vec{r},\vec{r}_0,\omega)&=-\mathcal{H}_{ij}^{(N)(1)}(\vec{r},\vec{r}_0,\omega),\\ \mathcal{G}_{ij}^{(N)(1)}(\vec{r},\vec{r}_0,\omega)&=+\tilde{\mathcal{G}}_{ij}^{(N)(1)}(\vec{r},\vec{r}_0,\omega).\end{aligned} \tag{7.13}$$

In particular on using (3.13) and (7.8), we have

$$\mathcal{E}_{xx}^{(N)(1)}(\vec{r},\vec{r}_0\omega)=-\hbar\eta(-\omega)(1+\coth\tfrac{1}{2}\beta\omega\hbar)\times\left(k_0^2+\frac{\partial}{\partial x^2}\right)\frac{\sin k_0R'}{R'}, \tag{7.14}$$

$$\mathcal{E}_{zz}^{(N)(1)}(\vec{r},\vec{r}_0\omega)=+\hbar\eta(-\omega)(1+\coth\tfrac{1}{2}\beta\omega\hbar)\times\left(k_0^2+\frac{\partial^2}{\partial z^2}\right)\frac{\sin k_0R'}{R'}, \tag{7.15}$$

and on taking Fourier transforms (7.14) and (7.15) lead to

$$\mathcal{E}_{xx}^{(N)(1)}(\vec{r},\vec{r}_0,t)=\frac{-4\hbar c}{\pi}\sum_{n=1}^{\infty}\frac{(\beta\hbar cn-ict)^2+2R_x'^2-R'^2}{[(\beta\hbar cn-ict)^2+R'^2]^3}, \tag{7.16}$$

$$\mathcal{E}_{zz}^{(N)(1)}(\vec{r},\vec{r}_0,t)=\frac{+4\hbar c}{\pi}\sum_{n=1}^{\infty}\frac{(\beta\hbar cn-ict)^2+2R_z'^2-R'^2}{[(\beta\hbar cn-ict)^2+R'^2]^3}. \tag{7.17}$$

The corresponding translationally invariant correlations $\mathcal{E}_{ij}^{(N)(0)}$ are given by (4.12). The black-body radiation is no longer isotropic as the translation invariance is broken.

One can now study temporal and spatial fluctuations of the black-body radiation in a manner similar to that of Mehta and Wolf. In the present case owing to breakdown of translational invariance, temporal coherence itself depends on the point $\vec{r}_0$, e.g.,

$$\mathcal{E}_{xx}^{(N)(1)}(\vec{r}_0,\vec{r}_0,t)=-\frac{4\hbar c}{\pi}\sum_{1}^{\infty}\frac{(\beta\hbar cn-ict)^2-4z_0^2}{[(\beta\hbar cn-ict)^2+4z_0^2]^3}, \tag{7.18}$$

$$\mathcal{E}_{zz}^{(N)(1)}(\vec{r}_0,\vec{r}_0,t)=+\frac{4\hbar c}{\pi}\sum_{1}^{\infty}[(\beta\hbar cn-ict)^2+4z_0^2]^{-2}. \tag{7.19}$$

In particular for $z_0\to 0$, we obtain

$$\mathcal{E}_{xx}^{(N)}\to 0,\quad \mathcal{H}_{zz}^{(N)}\to 0,\quad \mathcal{E}_{zz}^{(N)}\to 2\mathcal{E}_{zz}^{(N)(0)}=\frac{8\hbar c}{\pi(\hbar c\beta)^4}\times\zeta[4,1-ict/\hbar c\beta],\quad \mathcal{H}_{xx}^{(N)}\to 2\mathcal{H}_{xx}^{(N)(0)}, \tag{7.20}$$

where $\zeta[\mathrm{S}, a]$ is the generalized Riemann ζ function defined by[24]

$$\zeta[\mathrm{S}, a]=\sum_{\mathrm{S}=0}^{\infty}(n+a)^{-\mathrm{S}} . \quad (7.21)$$

We thus see that the magnitude of $\mathcal{E}_{zz}^{(N)}$, in the limiting case $z_0 \to 0$, is twice that in the absence of the conducting surface whereas $\mathcal{E}_{xx}^{(N)}$ vanishes. It also follows from (7.8) that any two orthogonal components of the electric field at the same space point $\vec{r}$ are uncorrelated, i.e.,

$$\mathcal{E}_{ij}^{(N)}(\vec{r}_0, \vec{r}_0, t)=0 \quad \text{if } i \neq j . \quad (7.22)$$

To discuss the spatial coherence we put $t=0$ in (7.16), (7.17), and (4.12). The resulting summations can be evaluated in closed form:

$$\mathcal{E}_{xx}^{(N)}(\vec{r}, \vec{r}_0, 0)=\left(\frac{\hbar c \pi^3}{2\alpha^4}\right)\left[\frac{1}{R_1^4}\left(A(R_1)+B(R_1)\frac{R_{1x}^2}{R_1^2}\right) - R_1 \to R_1'\right] , \quad (7.23)$$

where

$$R_1=(\pi/\alpha)R, \quad R_1'=(\pi/\alpha)R', \quad \alpha=\hbar c\beta ,$$

$$A(R)=-R\coth R-R^2\operatorname{csch}^2 R - 2R^3\operatorname{csch}^2 R\coth R+4 , \quad (7.24)$$

$$B(R)=3R\coth R+3R^2\operatorname{csch}^2 R + 2R^3\operatorname{csch}^2 R\coth R-8 . \quad (7.25)$$

$\mathcal{H}_{xx}^{(N)}(\vec{r}, \vec{r}_0, 0)$ is given by (7.25) with the sign of the terms involving R' changed. The mixed correlation is given by (7.10):

$$\mathcal{G}_{ij}^{(N)(1)}(\vec{r}, \vec{r}_0, t)=+\tilde{\mathcal{G}}_{ij}^{(N)(1)}(\vec{r}, \vec{r}_0, t) = \frac{8\hbar c i}{\pi}\epsilon_{ijl}R_l'(1-2\delta_{j3}) \times\sum_{1}^{\infty}\frac{\beta\hbar c n-ict}{[(\beta\hbar c n-ict)^2+R'^2]^3} . \quad (7.26)$$

We have so far considered only the second-order correlation functions of the black-body radiation. Since higher-order correlation functions can be expressed in terms of the second-order correlation functions,[25,26] this completes our study of the black-body fluctuations.

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[1]G. S. Agarwal, in *Tracts in Modern Physics*, edited by G. Höhler *et al*. (Springer, New York, 1974), Vol. 70.

[2]H. Haken, *Laser Theory* (Springer, New York, 1970), Vol. XXV/2C.

[3]C. L. Mehta and E. Wolf, Phys. Rev. 134, A1143 (1964); 134, A1149 (1964).

[4]L. Mandel and E. Wolf, Rev. Mod. Phys. 37, 231 (1965).

[5]For a recent work on this problem, see S. B. Lai, P. L. Knight, and J. H. Eberly, Phys. Rev. Lett. 32, 494 (1974).

[6]W. Heitler, *Quantum Theory of Radiation* (Oxford U. P., London, 1954, 3rd ed.

[7]C. K. Carniglia and L. Mandel, Phys. Rev. D 3, 280 (1971).

[8]The importance of the field propagator, in the spontaneous-emission problem has also been emphasized by R. Bullough, in *Coherence and Quantum Optics*, edited by L. Mandel and E. Wolf (Plenum, New York, 1973), p. 121.

[9]R. Kubo, Rep. Prog. Phys. 29, 255 (1966).

[10]P. C. Martin, in *Many Body Physics*, edited by C. Dewitt and R. Belian (Gordon and Breach, New York, 1968), p. 37.

[11]The case of systems described by more general density matrices, such as those relevant in the problem of relativistic statistical mechanics of black-body radiation [cf. J. H. Eberly and A. Kujawski, Phys. Rev. 155, 10 (1967)], will be discussed in a latter paper of this series.

[12]Cf. M. Born and E. Wolf, *Principles of Optics* (Pergamon, London, 1970).

[13]Similar relations were proved for the classical correlations functions by P. Roman and E. Wolf [Nuovo Cimento 17, 462 (1960)], who, however, used a different method.

[14]L. Mandel, Phys. Rev. 152, 438 (1966).

[15]L. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, New York, 1960), p. 366.

[16]For a discussion of angular spectrum of plane waves, see e.g., A. Banos, *Dipole Radiation in Presence of a Conducting Half Space* (Pergamon, New York, 1966).

[17]From now onwards we will denote the translationally invariant contribution of Sec. IV by $\chi_{ijEE}^{(0)}$ and the contribution due to the presence of the surface by $\chi_{ijEE}^{(1)}$.

[18]For an excellent review article on dispersion forces see I. E. Dzyaloshinskii, E. M. Lifshitz, and L. P. Pitaevskii, Adv. Phys. 10, 165 (1961).

[19]For an account of spatial dispersion, see for example, V. M. Agranovich and V. L. Ginzburg, *Spatial Dispersion in Crystal Optics and the Theory of Excitons* (Interscience, New York, 1966).

[20]G. S. Agarwal, D. N. Pattanayak, and E. Wolf, Phys. Rev. Lett. 27, 1022 (1971); Phys. Rev. B 10, 1447 (1974).

[21]A. A. Maradudin and D. L. Mills, Phys. Rev. B 7, 2787 (1973).

[22]J. L. Birman and J. J. Sein, Phys. Rev. B 6, 2482 (1972).

[23]This condition now replaces the mode-coupling conditions of Ref. 20 when retardation effects are ignored. The condition (6.9) acts as an additional boundary condition.

[24]E. Whittaker and G. N. Watson, *A Course in Modern Analysis* (Cambridge U.P., Cambridge, England, 1965), p. 266.

[25]Cf. with the moment theorem for Gaussian random processes: I. S. Reed, IRE Trans. Inf. Theory IT8, 194 (1962).

[26]Black-body fluctuations in finite domains have also been recently discussed by H. P. Baltes, E. R. Hilf, and M. Pabst, Appl. Phys. (Germany) 3, 21 (1974).