Mirror Effects on Dynamic Response of Surface-Emitting Lasers

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***Abstract*— The effects of longitudinal wave distribution and mirror structures on the small-signal response of surface-emitting lasers are analyzed for the first time. The analysis is based on an improved dynamic model implemented in the transfer-matrix representation. It is shown that for two structures with the same threshold gain and the same internal structure, high contrast mirrors give rise to a higher relaxation oscillation and modulation bandwidth for the same injection level.**

I. Introduction

Surface-emitting lasers are considered to be a promising device for optical communications and optical interconnect applications. So far, the analysis of the relaxation oscillation and the modulation response has been based on the conventional rate equations [1]. In surface emitting lasers, however, the gain medium constitutes only a small portion of the laser cavity, and the standing wave's spatial variation within the rather short cavity is substantial. Consequently, the use of the conventional photon rate equation becomes problematic as both the standing wave effect as well as the penetration of the field in the mirror must be properly accounted for [2]. Modifications to the conventional rate equation can be made by introducing an effective cavity length and photon life time. The problem, however, is only passed to another level. In this letter, we have developed a dynamic laser model based on a Green's function formalism following a first-principle model of Henry's [3]. To demonstrate the use of this model, we have applied it to the analysis of short-cavity lasers such as surface-emitting lasers. We show that two lasers that have the same threshold gain and the same internal structure can differ in the relaxation oscillation and the modulation response if high-contrast dielectric mirrors are deployed in one and semiconductor multilayer mirrors are utilized in the other. This demonstrates that the spatial variations of the field and gain can have significant impact on the lasers' dynamic responses.

II. Theoretical Model

Lasers with multilayer structure such as surface emission lasers can be conveniently treated using the transfer matrix method [4]. Starting from a semi-classical theory and using Green's function method [3]–[5], we obtain the following rate equation for the small signal analysis. In the transfer matrix representation of the L segments of an arbitrary structure, this new rate equation can be expressed as

$$\frac{d(\delta a/a_0)}{dt} = \sum_{k=1}^{L}(C_{Nkr}\delta N_k + C_{\mathrm{skr}}\delta S_k)l_k + F_r(t) + \sum_{k=1}^{L}\frac{\gamma\delta N_k}{2\tau_{\mathrm{sp}}I_0} \qquad (1)$$

where

$$C_{Xkr} = \mathrm{Re}\left[\frac{\beta_k Z_{Lk}^{+}(z_{k+1})Z_{Lk}^{-}(z_{k+1})(1+j\alpha_{Xk})\frac{dg_k}{dX_k}}{2\sum_{k=1}^{L} l_k(\frac{\beta_k}{v_{gk}})Z_{Lk}^{+}(z_{k+1})Z_{Lk}^{-}(z_{k+1})}\right] \qquad (2)$$

where δa is the time dependent deviation from the steady state value of the envelope amplitude a_0. Z_{Lk}^{+} and Z_{Lk}^{-} are the forward and backward propagating components of the field in the Kth segment. Z_{L1}^{+} and Z_{L1}^{-} satisfy the boundary conditions at the left end of the laser cavity and are related to those in the kth segment (Z_{Lk}^{+} and Z_{Lk}^{-}) through a transfer matrix [4]. β_k is the propagation constant and g_k is the modal gain in the k segment. $X = S, N$ (S is the photon density and N is the carrier density) while δN_k and δS_k are the deviations of the carrier density and the photon density from their stationary values respectively. l_k is the segment length, F_r is the real part of the Langevin noise force, α_X is the ratio between the real and imaginary parts of β_k due to the change in X_k. γ is the coupling coefficient of the spontaneous emission into the cavity mode while τ_{sp} is the spontaneous emission lifetime. v_{gk} is the group velocity defined as $v_{gk} = c/n_{gk}$ where c is the speed of light and n_g is the refractive index taking material dispersion into account, and subscript k denotes the Kth segment.

This new rate equation differs from that of [4], [5] in the following aspects. In previous work, the spontaneous emission contribution to the lasing mode R_{sp} is considered independent of the carrier density distribution in the structure [5]. In our analysis we consider $R_{\mathrm{sp}} = \frac{\gamma N}{\tau_{\mathrm{sp}}}$, consequently R_{sp} is a function of the carrier density and its small signal analysis becomes $\frac{\gamma\delta N}{2I_0\tau_{\mathrm{sp}}}a_0^2$ as seen in (1). This way the model covers the possible situation where the spontaneous emission is localized in the gain regions only. The expression for C_{Xkr} differs from that of [4] in that the propagation constant β_k and the group velocity v_{gk} are now locally variable from segment to segment rather than constants. In DFB structures the variation of refractive index along the laser structure is

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small. Accordingly, both β and v_g can be assumed constants. Such a simplification can lead to serious errors in the case of surface emitting lasers where high contrast mirrors (i.e., dielectric mirrors) are often used.

From the rate equation, we obtain the small signal modulation response

$$\frac{\delta a(\Omega)}{\delta a(\Omega=0)}=\frac{\Omega_r^2+\Gamma_N\Gamma_P}{(j\Omega+\Gamma_N)(j\Omega+\Gamma_p)+\Omega_r^2} \tag{3}$$

where $\Omega_r = 2\pi f_r, f_r$ is the relaxation oscillation frequency and is expressed as in (4), at the bottom of the page. The damping factors $\Gamma_{N,P}$ in (4) are expressed as

$$\Gamma_N=\left[\frac{\delta}{\delta N_m}(R_{\rm rm}+R_{\rm stm})\right] \tag{5}$$

where $R_{\rm rm}$ is the total nonstimulated recombination rate,

$$R_{\rm rm}=A_{\rm nr}N_m+BN_m^2+CN_m^3 \tag{6}$$

and $R_{\rm stm}$ is the stimulated emission rate,

$$R_{\rm stm}=v_g g_m S_{m0} \tag{7}$$

while

$$\Gamma_P=-2\sum_{k=1}^{L} C_{\rm skr}\, S_{k0} l_k \tag{8}$$

With the above improved model, one can now calculate the dynamic response of an arbitrary laser structure with nonuniform gain and field distributions that are typical in short cavity lasers.

As an example, we will examine two surface emitting lasers with identical internal structures but different mirror layers. Both structures have a gain region that contains five 10-nm thick GaAs quantum wells spaced by 111.2 nm of $Al_{0.15}Ga_{0.85}As$ [6]. The first structure has semiconductor mirrors that consist of a top multilayer reflector stack with 27 periods of $Al_{0.25}Ga_{0.75}As$-AlAs which are 61.4 and 71.1 nm thick, respectively, while the bottom mirror consists of 37.5 periods layers of the same composition. The threshold gain of such a structure is calculated to be 47.85/cm. While keeping everything else the same in the second structure, we employ dielectric mirrors and adjust the number of layers such that it would have the same threshold gain. Under this constraint, for a high/low index ZnS-SiO_2 cell of index ratio of 3.0/1.45 [7], the second structure has a top mirror of 5 periods while the bottom mirror has the same composition but 8.5 periods.

In Fig. 1, we compare the modulation responses of the two lasers at various injection levels. For the two mirror sets specified above, the difference in the relaxation oscillation frequency is around 35%. This difference would be zero in the framework of conventional rate equation [1], if the mode distribution inside the structure is not taken into account. To gain an insight into the physical cause behind such a large difference, a simplified expression for (4) can be obtained by assuming that the photon and carrier densities are constant inside each well. This assumption allows us to express f_r as shown in (9) at the bottom of the page where the subscript w denotes the well, and the subscript, avg denotes the average values, while L_D is the physical length of the entire device. As the injection current and the modal gain in both structures are set to be the same, the photon density in the well is also the same. If $N_w d_w$ was set equal to L_D in (10), and the term $\mathrm{Re}[\frac{\beta_w(1+j\alpha_{Nw})Z^-_{Lw}Z^+_{Lw}}{\beta_{\rm avg}Z^+_{L\rm avg}Z^-_{L\rm avg}}]$ was set to the average value of the group velocity, the above equation would become identical to the conventional expression for f_r, (e.g., (40) in [1]). It is thus clear that the use of the conventional rate equation always overestimates the relaxation oscillation frequency. Iga *et al.* [8] inserted a phenomenologic filling term $(N_w d_w/L_D)$ into the conventional rate equation for the case of surface emitting lasers. Such a correction would lead to an underestimation of the relaxation oscillation frequency as the term $\mathrm{Re}[\frac{\beta_w(1+j\alpha_{Nw})Z^-_{Lw}Z^+_{Lw}}{\beta_{\rm avg}Z^+_{L\rm avg}Z^-_{L\rm avg}}]$ in (5) is larger than unity and dependent on the specific laser.

In conclusion, we have shown that high contrast mirrors in surface-emitting lasers can lead to higher modulation response and relaxation frequency than low contrast ones. Such a result

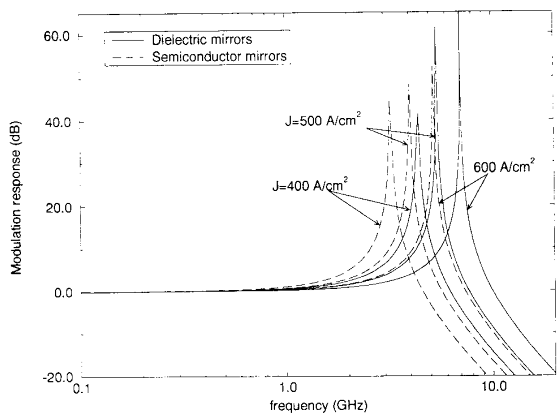

Fig. 1. Modulation response versus frequency at different injection levels for a typical surface emission laser with AlGaAs-GaAs mirror layers (solid lines) and for a laser of the same internal structure and the same threshold gain but with dielectric mirror layers (dashed lines).

$$f_r=\frac{1}{2\pi}\sqrt{2\sum_{m=1}^{N_w}\left[\mathrm{Re}\left(\frac{\beta_m Z^+_{Lm}(z_{m+1})Z^-_{Lm}(z_{m+1})(1+j\alpha_{Nm})\frac{dg_m}{dN_m}}{2\sum_{k=1}^{L} l_k(\frac{\beta_k}{v_{gk}})Z^+_{Lk}(z_{k+1})Z^-_{Lk}(z_{k+1})}\right)+\frac{\gamma}{2\tau_{\rm sp}I_0}\right]l_m v_{\rm gm} g_m S_{\rm m0}} \tag{4}$$

$$f_r=\frac{1}{2\pi}\sqrt{g_w\frac{dg_w}{dN}v_{gw}S_w\left(\frac{N_w d_w}{L_D}\right)\mathrm{Re}\left[\frac{\beta_w(1+j\alpha_{Nw})Z^-_{Lw}Z^+_{Lw}}{(\frac{\beta}{v_g}Z^+_L Z^-_L)_{\rm avg}}\right]+\frac{\gamma}{\tau_{\rm sp}}\frac{S_w g_w v_g N_w d_w}{S_{\rm avg}L_D}} \tag{9}$$

is consistent with the predictions of the modified rate equations if the photon lifetime (or effective length) is correctly calculated and incorporated [2]. Notice that this effect arises from the modal distribution difference in the structures, and is automatically accounted for in this model without introducing an effective cavity length or a standing wave factor. For the structures analyzed in this letter the high contrast dielectric mirrors result in an increase in bandwidth of 35% over that of low contrast mirrors made of semiconductors. The present generalized model can readily be applied to the standard FP and DFB lasers as special cases where one can verify that (1), (2), and (5) automatically reduce to the conventional rate equations.

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